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A NEW FILTER ERROR METHOD APPLIED TO AIRCRAFT PARAMETER ESTIMATION

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RESUMO
A new filter method is presented and tested for the estimation of the aerodynamic coefficients of aircraft. The proposed method uses a stochastic approach to obtain the parameters estimates from the state variables filtering errors. This supports an approach where the state variable filtering problem and the parameter estimation problem are treated separately resulting into two filtering problems of smaller dimensions and less computational burden. Test results indicate that the method is capable of providing good aerodynamic coefficients estimations.

PALAVRAS CHAVES
Aerodynamic Coefficients; Aerodynamic Modeling; Kalman Filter; Parameter Estimation.

1. INTRODUÇÃO

The number of multiplications necessary to compute the error covariance matrix used in Kalman based filters generally varies with the third power of the state vector dimension (Gelb, 1976). In problems involving the estimation of system parameters it is usual to generate extended state vectors of large dimension, due to the fact that the dynamic of these parameters must be incorporated into de state variable dynamics (e.g., Curvo, 2000). Much effort has been undertaken in attempts to develop techniques to minimize the computational burden imposed by the problem of parameter estimation and most of these techniques are aimed at reducing the filter size. Often, after the elimination of some of the variables the system reaches an irreducible minimum and the filter still makes excessive demands on the computer. An alternative technique is to break the high-order filter into mutually exclusive lower-order filters, each with separated covariance calculations. Considering the numerical complexity condition stated above, the advantage of breaking a filter into filters of smaller dimensions becomes evident.

The method presented, adopts a decoupling scheme where the parameter estimation is treated separately from the state estimation, resulting in reduced numerical complexity. The method is intended to be applicable in on board real time or near real time identification of time invariant nonlinear dynamic systems, with process and measurement noise, and still have the inherent stabilizing properties of usual filter error methods (Jategaonkar and Thielecke, 1994).

Though significant advances have been made in the estimation of aircraft dynamical model parameters (Greenberg, 1951; Klein, 1981; Maine and Iliff, 1985; Klein, 1989; Jategaonkar and Plaestschke, 1989; Jategaonkar and Thielecke, 1994, 2000; Bauer, 1990; Curvo 2000, 2002), there is much to be explored in the development of on board real or near real time estimation of aerodynamic parameters, increasing safety and reducing the overall development costs of flight testing necessary for aircraft specification and certification. There is also the opportunity of further exploring the advancements in modern control theory to implement adaptive flight controllers to get improved performance. The method presented has the required characteristics to attend the demands of these opportunities, combining the realism of nonlinear work models, with measurement and process noise at the cost of a low numerical complexity that allows real time implementation.

The paper is organized as follows: section 2 presents the model formulation for system identification, section 3 presents the proposed method; in section 4, an example of identification of the longitudinal dynamic model of a generic aircraft is demonstrated and preliminary results presented, and finally in Section 5 conclusions are drawn concerning method characteristics and test results.

2. MODEL FORMULATION

The dynamic system, whose parameters are to be estimated, is modeled by the following non-linear stochastic equations:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{p}, t) + \boldsymbol{\omega}(t) \qquad \mathbf{x} \in \Re^n \tag{1}$$

$$\mathbf{y}(t_k) = \mathbf{h}(\mathbf{x}(t_k), t_k) + \boldsymbol{\nu}_x(t_k) \qquad \mathbf{y} \in \Re^p \tag{2}$$

The functions $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are non-linear real valued vector functions. These functions are assumed to have sufficient differentiability in order to allow a Taylor series expansions. The process and measurement noise $\boldsymbol{\omega}(\mathbf{t})$ e $\boldsymbol{\nu}_\mathbf{x}(\mathbf{t_k})$ are assumed to be zero mean Gaussian white noise processes, that is: $\boldsymbol{\omega}(\mathbf{t}) = \mathbf{N}(\mathbf{0}, \mathbf{Q_x})$ and $\boldsymbol{\nu}_\mathbf{x}(\mathbf{t_k}) = \mathbf{N}(\mathbf{0}, \mathbf{R_x})$. The matrices $\mathbf{Q}_x$ and $\mathbf{R_x}$ are the noise spectral density and covariance matrices. Furthermore, process and measurement noises are assumed to be independent. The parameter vector, $\mathbf{p}$, is to be estimated from measurements of system response $\mathbf{y}(\cdot)$ to given inputs $\mathbf{u}(\cdot)$ based on the system model postulated in Eqs. (1) and (2).

3. IDENTIFICATION METHOD.

In the method presented herein the unknown parameters are estimated from predicted output, in a scheme (Fig. 1) where previously filtered values of the state vector are used. The required predicted response is calculated using the filtered state variables and a numerical integration algorithm. Here a first order Euler integrator was adopted.

$$\hat{\mathbf{x}}(t+\Delta t) \cong \hat{\mathbf{x}}(t) + \mathbf{f}\left(\hat{\mathbf{x}}(t), \mathbf{u}(t), \mathbf{p}, t\right)\Delta t \tag{3}$$

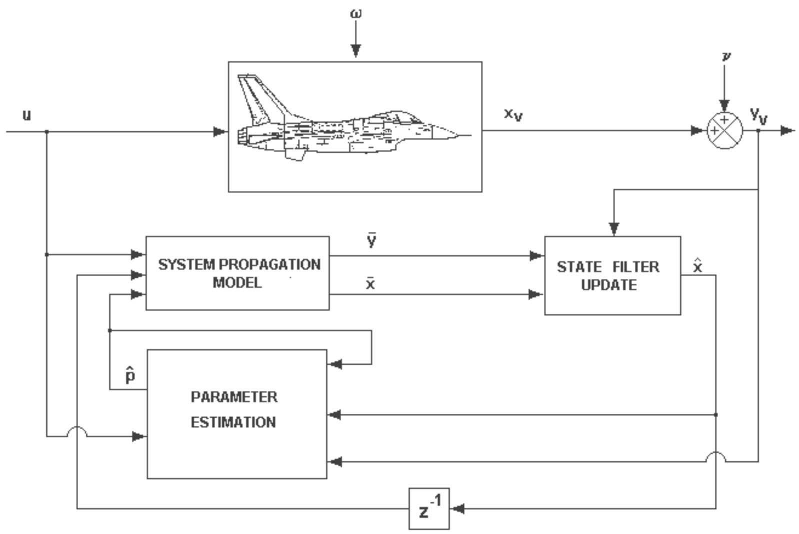


Fig. 1 – Scheme for Parameter Identification.

Assuming small errors in state vector estimation, a first order expansion of the output equation, Eq. (2), around the propagated value of previously filtered estate vector can be taken to model the system predicted response:

$$\mathbf{y}(t+\Delta t) \cong \mathbf{h}\left(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t\right) + \boldsymbol{\nu}_p(t+\Delta t) \tag{4}$$

where

$$\boldsymbol{\nu}_p(t+\Delta t) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(t+\Delta t)} \hat{\mathbf{e}}_x(t+\Delta t) + \boldsymbol{\nu}_x(t+\Delta t) \tag{5}$$

and by definition:

$$\hat{\mathbf{e}}_x(t+1) \equiv \mathbf{x}(t+\Delta t) - \hat{\mathbf{x}}(t+\Delta t)$$

$$E\{\hat{\mathbf{e}}_x(t+\Delta t)\} \equiv 0$$

$$\mathrm{Cov}\{\hat{\mathbf{e}}_x(t+\Delta t)\} \equiv \hat{\mathbf{P}}(t+\Delta t)$$

The following expression for calculating predicted output error then results from the expansion of $\mathbf{h}(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t)$, Eq. (4), into a Taylor series around the previously known parameter vector, $\overline{\mathbf{p}}$.

$$\mathbf{y}(t+\Delta t) \cong \mathbf{h}(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t)\Big|_{\mathbf{p}=\overline{\mathbf{p}}} + \left[\frac{\partial \mathbf{h}(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t)}{\partial \mathbf{x}} \frac{\partial \hat{\mathbf{x}}(t+\Delta t)}{\partial \mathbf{p}}\right]\Bigg|_{\mathbf{p}=\overline{\mathbf{p}}} \Delta\mathbf{p} + \boldsymbol{\nu}_p(t+\Delta t) \tag{6}$$

Rearranging the terms, and adopting a compact notation, Eq. (6) may be put into the following form:

$$\Delta\mathbf{z}(t+\Delta t) = \mathbf{H}(t+\Delta t)\Delta\mathbf{p} + \boldsymbol{\nu}_p(t+\Delta t) \tag{7}$$

where

$$\Delta\mathbf{z}(t+\Delta t) = \mathbf{y}(t+\Delta t) - \mathbf{h}(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t)\Big|_{\mathbf{p}=\overline{\mathbf{p}}}$$

$$\mathbf{H}(t+\Delta t) = \left[\frac{\partial \mathbf{h}(\hat{\mathbf{x}}(t+\Delta t), t+\Delta t)}{\partial \mathbf{x}} \cdot \frac{\partial \hat{\mathbf{x}}(t+\Delta t)}{\partial \mathbf{p}}\right]\Bigg|_{\mathbf{p}=\overline{\mathbf{p}}}$$

The problem can now be taken as one of estimating $\Delta\mathbf{p}$ from the observation predicted output error (Eq. (7)), given the a priori information:

$$\Delta\mathbf{p} + \mathbf{e}_p = 0, \ \mathrm{cov}\{\mathbf{e}_p\} \equiv \overline{\mathbf{P}}_p \tag{8}$$

where $\overline{\mathbf{P}}_p$ is the covariance matrix of the a priori errors in the parameters and the matrix $\mathbf{R}_p(t+\Delta t)$, is defined as:

$$\mathrm{cov}\{\boldsymbol{\nu}_p(t+\Delta t)\} \equiv \mathbf{R}_p(t+\Delta t)$$

and calculated from Eq. (5) such that

$$\mathbf{R}_p(t+\Delta t) = \left[\left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(t+\Delta t)}\right]^T \mathbf{P}_x(t+\Delta t)\left[\left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(t+\Delta t)}\right] + \mathbf{R}_x \tag{9}$$

where:

$$\mathbf{P}_x(t+\Delta t) \equiv \mathrm{cov}\{\hat{\mathbf{e}}_x(t+\Delta t)\}$$

$$\mathbf{R}_x \equiv \mathrm{cov}\{\boldsymbol{\nu}_x(t+\Delta t)\}$$

$$E\{\hat{\mathbf{e}}_x(t+\Delta t)\,\boldsymbol{\nu}^T{}_x(t+\Delta t)\} \equiv 0$$

The solution of the problem, that is, the estimated value of $\Delta\mathbf{p}$ is then found through a stochastic optimal linear estimator with a priori information applied to the problem described by Eqs. (8) and (7) (e.g., Gelb, 1976).

$$\Delta\hat{\mathbf{p}} = \Delta\overline{\mathbf{p}} + \left(\overline{\mathbf{P}}_p{}^{-1} + \mathbf{H}^T(t+\Delta t)\mathbf{R}_p{}^{-1}(t+\Delta t)\mathbf{H}(t+\Delta t)\right)^{-1}\mathbf{H}^T(t+\Delta t)\mathbf{R}^{-1}{}_p{}^T(t+\Delta t)\left(\Delta\mathbf{z}(t+\Delta t) - \mathbf{H}(t+\Delta t)\Delta\overline{\mathbf{p}}\right) \tag{10}$$

Since $\Delta\overline{\mathbf{p}} \equiv \Delta\mathbf{p} + \mathbf{e}_p = 0$, then:

$$\Delta\hat{\mathbf{p}} = \mathbf{P}_p(t+\Delta t)\mathbf{H}^T(t+\Delta t)\mathbf{R}^{-1}{}_p{}^T(t+\Delta t)\Delta\mathbf{z}(t+\Delta t) \tag{11}$$

where

$$\mathbf{P}_p(t+\Delta t) = \left(\overline{\mathbf{P}}_p{}^{-1} + \mathbf{H}^T(t+\Delta t)\mathbf{R}_p{}^{-1}(t+\Delta t)\mathbf{H}(t+\Delta t)\right)^{-1} \tag{12}$$

Notice that the estimated value, $\Delta\hat{\mathbf{p}}$, can also be seen as the $\Delta\mathbf{p}$ that minimizes the following quadratic form:

$$J(\mathbf{p}) = \frac{1}{2}\left[(\Delta\mathbf{p} - \Delta\overline{\mathbf{p}})^T \overline{\mathbf{P}}_p^{-1}(\Delta\mathbf{p} - \Delta\overline{\mathbf{p}}) + (\Delta\mathbf{z}(t+\Delta t) - \mathbf{H}(t+\Delta t)\Delta\mathbf{p})^T \mathbf{R}_p^{-1}(t+\Delta t)(\Delta\mathbf{z}(t+\Delta t) - \mathbf{H}(t+\Delta t)\Delta\mathbf{p})\right] \tag{13}$$

To reiterate in $t = t + \Delta t$ it is sufficient to define, observing that $\Delta\hat{\mathbf{p}} = \hat{\mathbf{p}} - \overline{\mathbf{p}}$ or $\hat{\mathbf{p}} = \overline{\mathbf{p}} + \Delta\hat{\mathbf{p}}$:

$t+\Delta t$		t
$\overline{\mathbf{p}}$	$\equiv$	$\hat{\mathbf{p}}$
$\overline{\mathbf{P}}_p$	$\equiv$	$\mathbf{P}_p(t)$

4. EXAMPLE

The example problem adopted for demonstration of the proposed technique consists of a simplified longitudinal dynamic model of a generic aircraft. The model is described by a set of non-linear ordinary differential equations (Etkin, 1972; Nelson, 1990; Durham, 1998) and with the same geometric, mass and aerodynamic data as in Nelson (1990) and Curvo (2000). The aerodynamic model is a classical nonlinear longitudinal model most commonly used in simulations, performance, and stability and control studies (Etkin, 1972; Nelson, 1990; Durham, 1998). Some simplifying assumptions were considered, namely: compressibility effects, and density variations due to velocity and altitude changes are not taken into account. Simulated data is used to estimate the following aerodynamic derivatives:

CL_0 , CL_α, $CL_{\dot\alpha}$, CL_{δ_e}, Cm_0, Cm_α, $Cm_{\dot\alpha}$, Cm_q and Cm_{δ_e}.

The use of simulated data does not invalidate the results. The objective is to test and demonstrate the procedure. Here, contrary to common practice, the unsteady aerodynamics effects, which depend on $\dot\alpha$, are not combined with the aerodynamic effects due to q. This is generally done in order to alleviate identification difficulties cause by the fact that $\dot\alpha$ and q are nearly the same for most maneuvers.

The state vector, is defined by the following set of variables:

$$\mathbf{x}^T = [V \quad \alpha \quad q \quad \theta \quad h] \tag{14}$$

Plant dynamic disturbances ($\boldsymbol{\omega}$) and measurement noise were included in the simulations. The plant disturbance is modeled using the Von Karman model for atmospheric turbulence (Nelson, 1990). Simulated measurement noise ($\mathbf{v_x}$) is considered to be white and gaussian with zero mean and variance $\mathbf{R}_x$. Results for *simulated measurements*, *estimated values*, and *true values* of the aircraft response are shown in Figures (3) through (4) and the control input in Figure (5). The simulations using the estimated coefficients (estimated values) are in good agreement with the simulations using the true values.

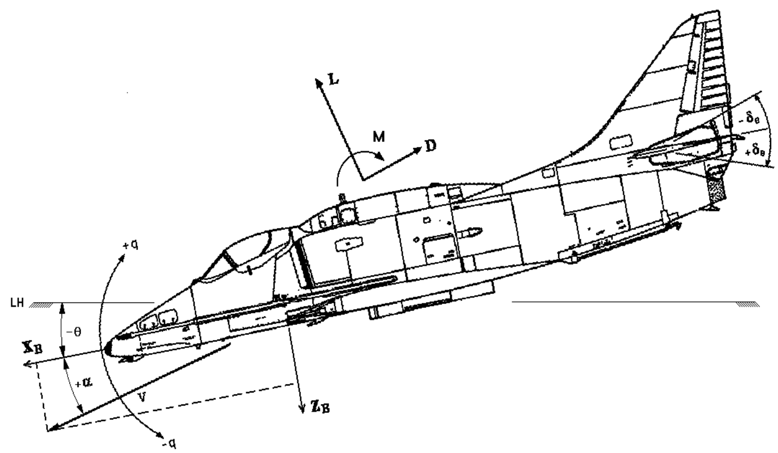
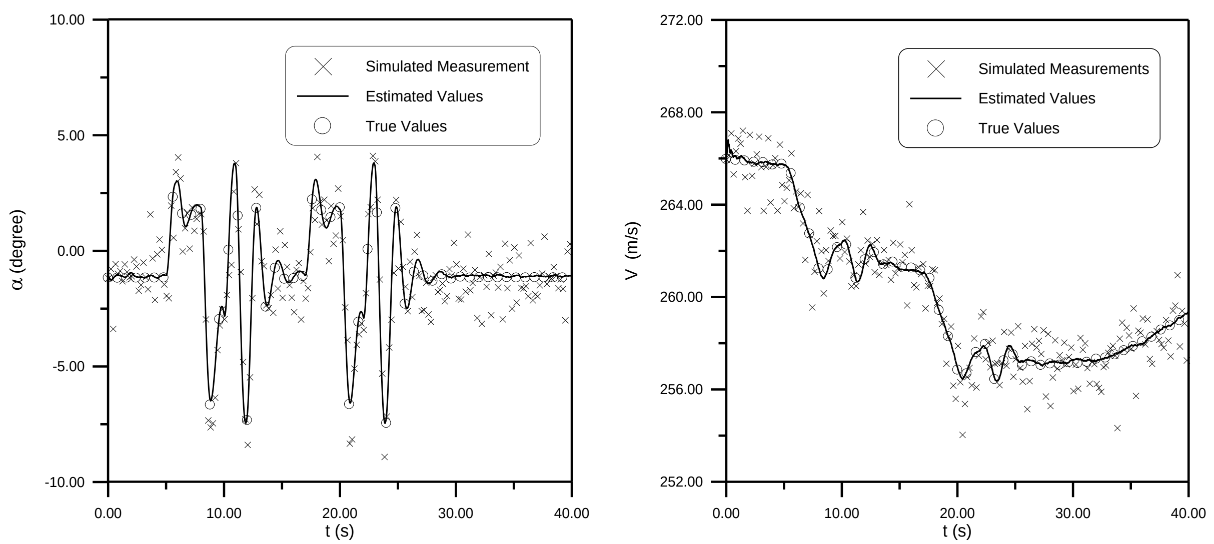
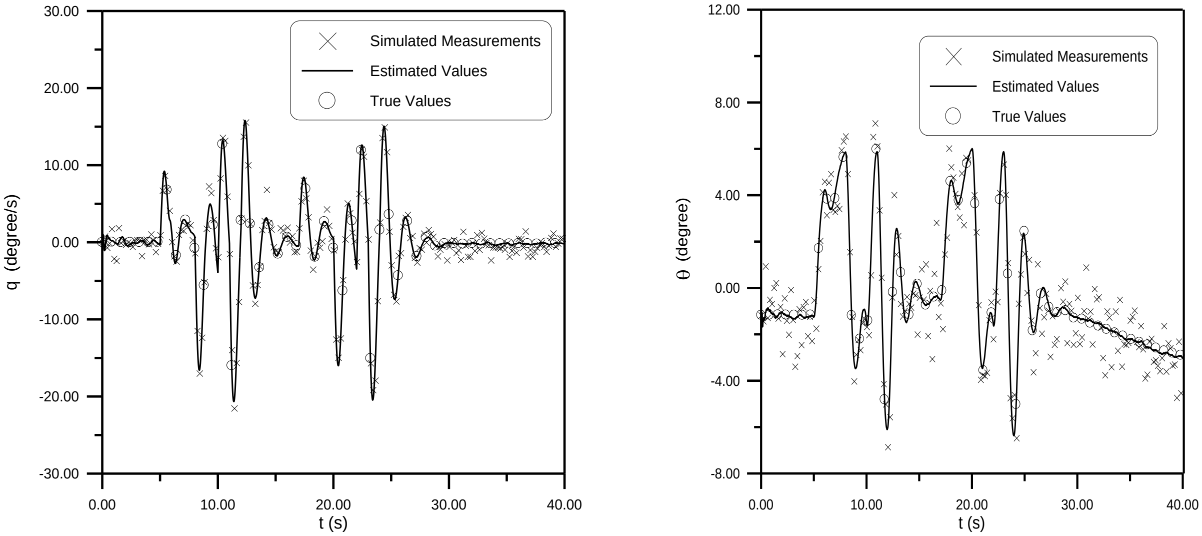


Fig.2 – Parameters of the Longitudinal Dynamics.



(a) Airspeed. (b) Angle of Attack.

Fig. 3 – Time History of State Variables.



(a) Pitch Rate. (b) Angle of Attitude.

Fig. 4 – Time History of State Variables.

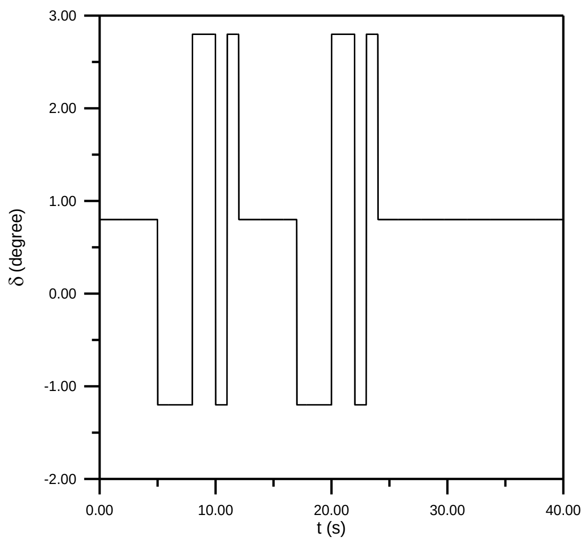
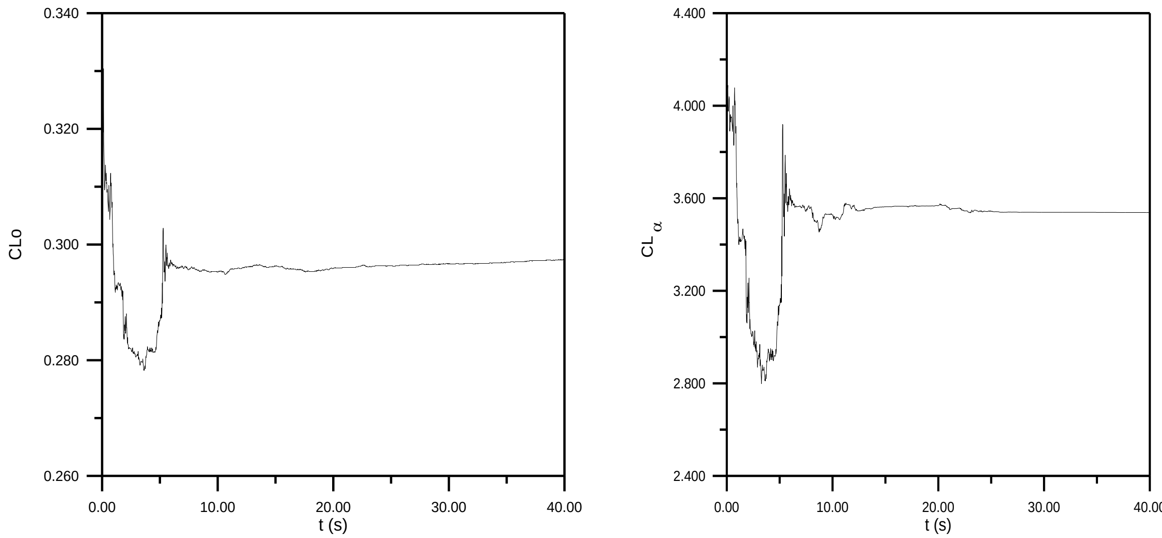


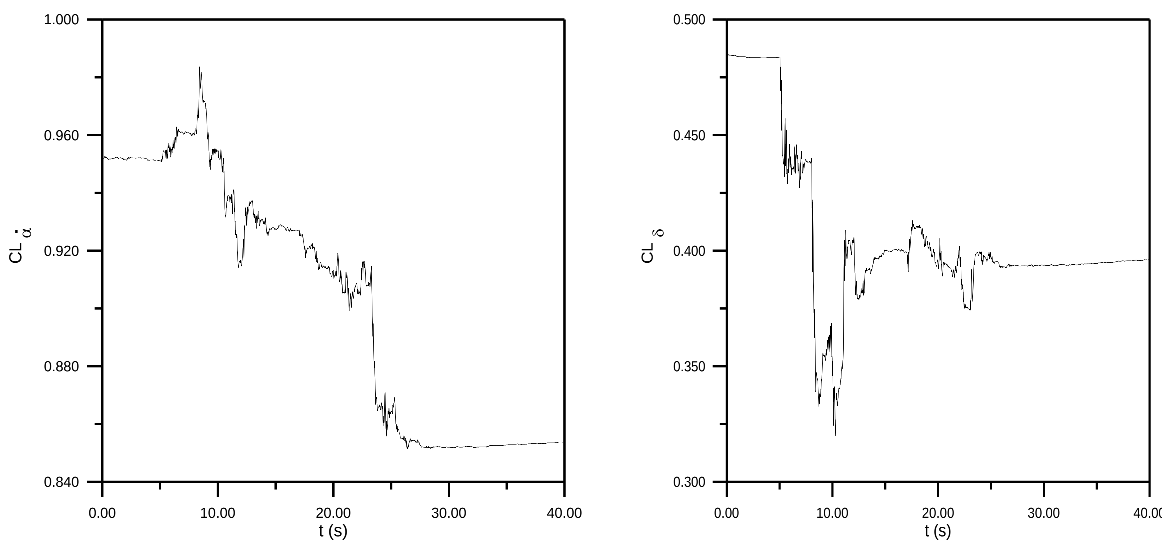
Fig. 5 – Applied Control Sequence (elevator deflection).

Figures (6) through (11) show the time histories for de estimated aerodynamic derivatives. In all cases, the derivative values settle almost immediately after the maneuver. These results are encouraging. If the time histories of these parameters kept varying even after the maneuver, through the end of the estimation time history, then little confidence could be held in the estimation of these coefficients.



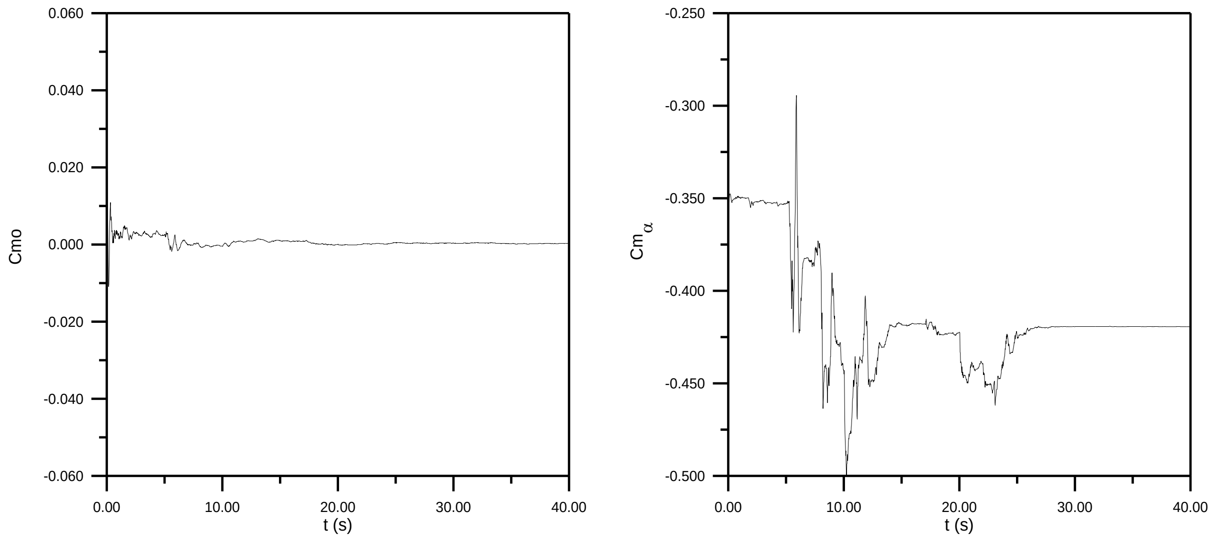
(a) Independent Term. (b) Derivative due to Angle of Attack.

Fig. 6 – Time History of Lift Coefficient.



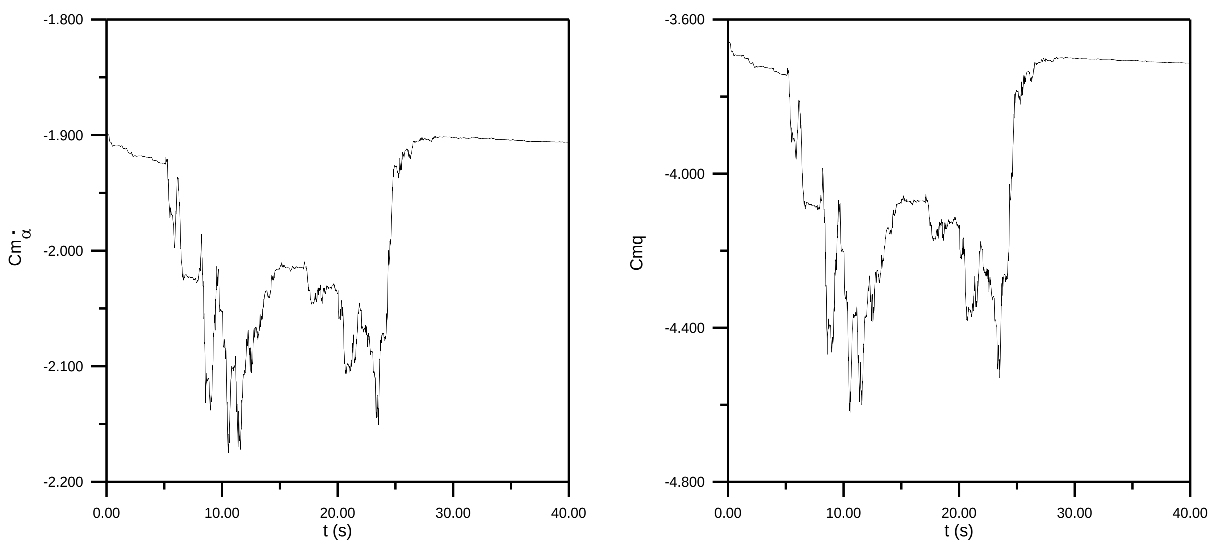
(a)Derivative due to Angle of Attack Rate. (b)Derivative due to Elevator Deflection.

Fig. 7 – Time History of the Lift Coefficient



(a) Independent Term. (b) Derivative due to Angle of Attack.

Fig. 8 – Time History of Pitching Moment Coefficient.



(a) Derivative due to Angle of Attack Rate. (b) Derivative due to Pitch Rate.

Fig. 9 – Time History of Pitching Moment Coefficient.

For the purpose of comparison, the estimated aerodynamic coefficients are summarized in Table 1. At the same table, the true values used for simulations as well as the initial values for the estimation problem are presented. In the example problem, the coefficients used to initialize the estimation processes are equal to the values used for simulation purposes, called *true values*, corrupted by approximately ±15%, called *initial values*.

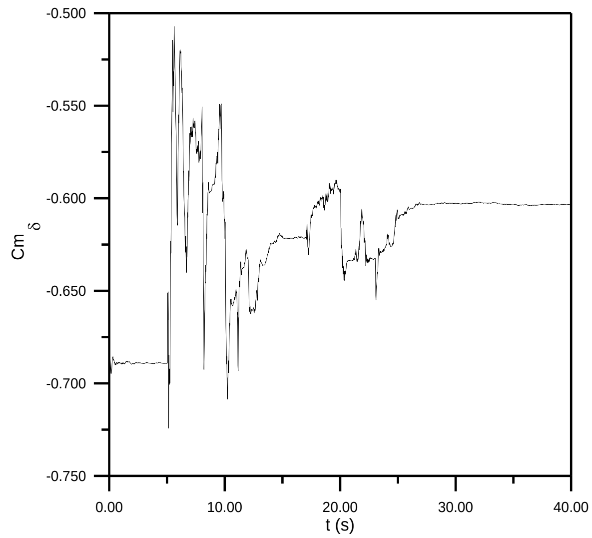


Fig. 10 – Time History of Pitching Movement Derivative due to Elevator Deflection.

Table 1 – Parameter Estimates for Simulated Data.

	True Values	**Initial Values (Simulation)**	**Estimated Values (Estimation)**
CL_0	0.30000	0.34500	0.29741 ±0.00065
CL_α	3.45000	3.98600	3.54006 ±0.02088
$CL_{\dot{\alpha}}$	1.20000	0.95200	0.85384 ±0.23322
$CL_{\delta e}$	0.40000	0.48600	0.39602 ±0.03628
Cm_0	0.00000	0.00000	0.00038 ±0.00078
Cm_α	-0.41000	-0.34900	-0.41918 ±0.02363
$Cm_{\dot{\alpha}}$	-1.65000	-1.89800	-1.90584 ±0.48069
Cm_q	-4.30000	-3.65500	-3.71197 ±0.77615
$Cm_{\delta e}$	-0.60000	-0.69000	-0.60317 ±0.04429

One of the most important aspects of estimation is the modeling of noise (disturbances) inherent to the processes. The statistical data used for estimation are shown in Tables 2 and 3. These tables contain data related to the initial state and parameter variance ($\mathbf{P_o}$), plant disturbance ($\mathbf{Q}_x$), and measurement errors ($\mathbf{R}_x$). The initial values for the variance of the parameters to be estimated followed a criterion suggested by Bauer and Andrisani (1990), where $P_o \approx diag[(0.25\,\bar{p}_j)^2, j = 1,2,...,n_p]$. For parameter values equal to or close to zero, the initial variance is established based on engineering judgment. The initial values for the state variables variance were assumed to be the same as the measurement noise variance. The specification of variance for state vector (V, α, q, θ, h) and measurement noise is the most troublesome characteristic in estimation problems. Here both are assumed to be white with intensities given by $\mathbf{Q}_x$ and $\mathbf{R}_x$, respectively. Process noise ($\mathbf{Q}_x$) the most difficult to determine. For this application, $\mathbf{Q}_x$ was chosen to be small enough, based on previous experience, and then adjusted using the residuals as guideline (filter tuning) in order to obtain the best filter performance. $\mathbf{R}_x$ is obtained directly from the measured data.

Table 2 – Initial Process Variance, Noise Variance.

State/Parameter	$\mathbf{P_0}$	$\mathbf{Q}_x$
V_m	1.0000E0	0.5786D-2
α_m	1.0000E0	0.8416D-2
q_m	1.0000E0	0.2692D-2
θ_z	1.0000E0	0.2990D-3
h	4.0000E0	0.4039D+3
CL_0	5.077E-3	
CL_α	9.930E-1	
CL_α	5.664E-2	
$CL_{\delta e}$	1.323E-2	
Cm_0	1.000E-4	
Cm_α	7.613E-3	
Cm_α	2.525E-1	
Cm_q	8.349E-1	
$Cm_{\delta e}$	2.976E-2	

Table 3 – Measurement Variance.

Measurement	$\mathbf{R}_x$
V_m	1.000E0
α_m	1.000E0
Q_m	1.000E0
θ_z	1.000E0
H	4.000E0

5. CONCLUSIONS

A new stochastic output error method, for parameter identification, was presented and demonstrated using the aerodynamic longitudinal model of a generic aircraft. The results obtained are encouraging. The method is simple enough to provide computing algorithms that can be packed as a subroutine and included in flight computers, so that the estimation can be conducted in near real time during flight tests. Since the method is of the filter error type it can naturally deal with unavoidable modeling errors, which can be taken into account by dynamic compensation with colored process noise, at the cost of few extra parameters to be estimated.

The estimated values were coherent with the real values. The estimates behavior, however, are mixed in character. Some parameters are easier to estimate than others; this behavior seems to be directly linked to the quality of the observation data. As in most cases the dynamic coefficients, $Cm_{\dot{\alpha}}$ and Cm_q are the most troublesome to be estimated, especially when they are not combined into a single coefficient.

6. ACKNOWLEDGEMENT

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