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A Stochastic Rudder Control Law for Ship Path-following Autopilots*

ATAIR RIOS-NETO† and JOSÉ JAIME DA CRUZ‡

An algorithm for generating a discrete-time stochastic adaptive control law in the presence of model uncertainties is shown to be potentially useful for ship path-following during navigation in restricted waters against current and wind perturbations.

Key Words—Ship control; adaptive control; stochastic control; state estimation; Kalman filters; nonlinear control systems; suboptimal control.

Abstract—A heuristic stochastic rudder control law for ship automatic path-following in restricted waters is presented. The objective is to make feasible an automatic pilot with an adaptive capacity of operating in situations where there is a great lack of knowledge of ship dynamics. State estimation and feedback control determination are separately treated. The extended Kalman filter is combined with a dynamical model compensation technique and a state noise adjustment procedure, in a situation where the observations have a high local level of information. An adaptive scheme results where unmodeled effects are estimated together with the system state. A feedback control law that uses state estimation information and steering conditions is proposed. In each discretization interval it takes a stepwise approximation for the control, as the solution of a parameter estimation problem. The performance of the resulting state estimator and controller is illustrated by results obtained through digital simulation. The test case selected is that of ship automatically steering in a channel under severe geometric and environmental conditions.

1. INTRODUCTION

MODELING of ship dynamics, incorporating all real world conditions of operation is a very complex or even an impossible task. In the case of maneuvering and steering in restricted waters, the limitations are mainly due to: (i) errors in the hydrodynamic derivatives; (ii) suction forces towards the nearest margin in channel operation; (iii) interference between close ships; and (iv) perturbations of wind and currents which can only be approximately evaluated as functions of ship parameters and environmental conditions.

In this context, the problem of rudder control is considered for ship automatic steering along prescribed paths. Based upon concepts of modern control theory (Jazwinski, 1970; Bryson and Ho, 1975; Gelb *et al.*, 1977), procedures for a state estimator and a controller, leading to an adaptive stochastic control scheme, are proposed (Rios-Neto and Cruz, 1979; Cruz, 1981). The objective is to have an automatic pilot (Fig. 1) that keeps an adaptive capability of automatic steering. It is designed to perform well in spite of substantial lack of knowledge of ship dynamics, as is the case during navigation in restricted waters under perturbation of winds and currents.

The extended Kalman filter (Jazwinski, 1970; Gelb *et al.*, 1977) is used to obtain estimates from measurements of range and range-rate from the ship to points of known coordinates, together with measurements of yaw angle, yaw-rate and rudder angle. To guarantee the quality of estimates, at each measurement time the high level of information

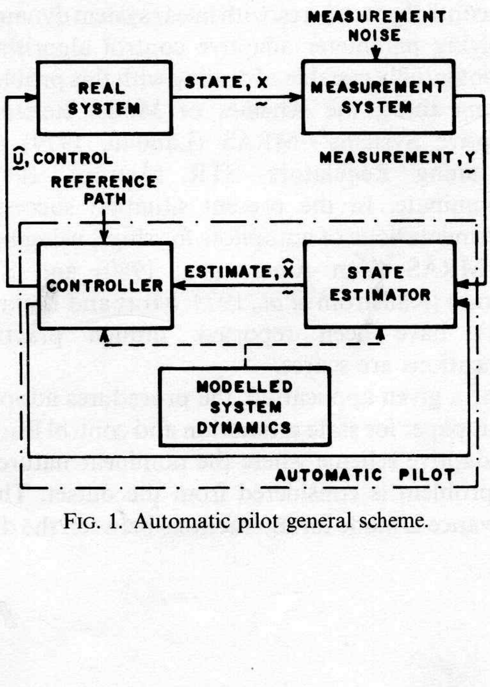

FIG. 1. Automatic pilot general scheme.

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† Instituto de Pesquisas Espaciais (Institute for Space Research), Conselho Nacional de Desenvolvimento Científico e Technológico (National Council for Scientific and Technological Development), C.P. 515,12200, São José dos Campos, SP, Brazil.

‡ Instituto de Pesquisas Tecnológicas (Technological Research Institute—Ship and Ocean Research Division), C.P. 7141, 01000 São Paulo, SP, Brazil.

contained in the measurement sets is combined with an adaptive scheme, to establish properly the state noise level (Gelb *et al.*, 1977) and with a dynamical model compensation technique (Tapley and Ingram, 1973; Cruz and Rios-Neto, 1980), to include effects not explicitly considered in the dynamics of the estimator. These unmodeled effects are locally represented by a first-order Gauss–Markov process which is then identified along with the estimation of ship state vector.

A heuristic procedure is proposed to establish the control law. The problem of the correction in rudder action at each discrete time is reduced to one of parameter estimation (Rios-Neto and Cruz, 1979). Using the information given by the state estimator, the control problem is separately solved. Guidance conditions are generated by predicting one interval ahead and constraining the rudder correction to take the state closer to the prescribed reference path and, with favorable kinematic conditions (in terms of direction and velocity), to follow that path. This results in an observation relationship of vector type. This pseudoobservation is corrupted by noise due to the state estimation error and also due to a stochastic interpretation of the linearization error. The latter is due to the high-order truncation errors resulting from the linear perturbation of the guidance conditions around the previous values of the control and of the state estimate. The rudder correction is then determined using a least-squares fit.

The problem associated with the automatic control of ship motion is certainly one where adaptive procedures are necessary in many situations (Åström, 1980a). In view of the complexity of dealing with realistic nonlinear system dynamic models, nonadaptive control systems are not capable of accommodating the varying conditions of operation and of environmental perturbations. Recent publications in adaptive control (Parks *et al.*, 1980; Jacobs, 1981; Isermann, 1982) indicate that control procedures, with linear system dynamics involving parameter adaptive control algorithms, are potentially capable of dealing with this problem. Among these, the schemes of Model Reference Adaptive Systems—MRAS (Landau, 1979) and Self-tuning Regulators—STR (Åström, 1980b) predominate. In the present situation successful implementations of autopilots for ships, using both the MRAS (Van Amerongen, 1980) and STR schemes (Källström *et al.*, 1979; Mort and Linkens, 1981), have been reported, though practical applications are scarce.

For a given application, the procedures adopted in this paper for state estimation and control lead to an adaptive scheme where the nonlinear nature of the problem is considered from the outset. Thus, allowance is made for interactions between the dual functions of estimation and control (Jacobs, 1981). As a consequence, the algorithms used in the design can give rise to a stochastic problem which is neither neutral nor separable. In terms of structure, following Parks *et al.* (1980), the corresponding adaptive control system can be classified as one with closed-loop adaptation (Fig. 1), since the effect of controller modification is fed back, affecting its adaptive adjustment.

To evaluate the performance of the proposed scheme, tests were carried out for a Mariner class vessel using digital simulation. The model considered to simulate observations of the real system planar motion includes hydrodynamic derivatives up to third order together with approximations for the effects of wind, current and suction toward nearest channel margin (Abkowitz, 1969; Strøm-Tejsen, 1965; Leone, Sotelo and Cruz, 1973; Chislett and Strøm-Tejsen, 1965). The good performance of both state estimator and controller is illustrated by the quality of results presented, which correspond to a ship maneuvering in a channel, under severe geometric and environmental conditions.

2. REAL SYSTEM MOTION SIMULATION MODEL

Digital simulation was adopted for preliminary testing of the feasibility of the proposed procedures in ship automatic piloting. Thus, a mathematical representation of the motion of the real system was necessary. The approach taken for modeling of ship dynamics was the one based on hydrodynamic derivatives (Abkowitz, 1969; Norrbin, 1970). The model adopted considers hydrodynamic derivatives up to third order with the possibility of including the perturbations of winds, current and suction toward nearest channel margin (Cruz, 1981; Strøm-Tejsen, 1965; Leone, Sotelo and Cruz, 1973; Chislett and Strøm-Tejsen, 1965). For wind and current perturbations, coarse approximations were made which are considered as reasonable for simulating their order of magnitude and qualitative behavior (Leone, Sotelo and Cruz, 1973). Roll (heel) and pitch motions were neglected (Strøm-Tejsen, 1965), thus restricting the validity of the model to situations where these motions have little influence on steering and maneuvering. The resulting planar equations of motion, following Newton's laws, in a body fixed coordinate system (O, x, y, z) parallel to the principal axes of inertia, but with the origin not necessarily at the center of gravity (Fig. 2), are expressed by (see Appendix):

$$F_{hx} + F_{cx} + F_{wx} = m\,(\dot{u} - rv - x_G r^2), \qquad (1)$$

$$F_{hy} + F_{cy} + F_{wy} = m\,(\dot{v} + ru + x_G \dot{r}), \qquad (2)$$

$$M_h + M_c + M_w = I_z \dot{r} + m x_G\,(\dot{v} + ru), \qquad (3)$$

where F_{hx}, F_{hy}, M_h together with F_{cx}, F_{cy}, M_c are the

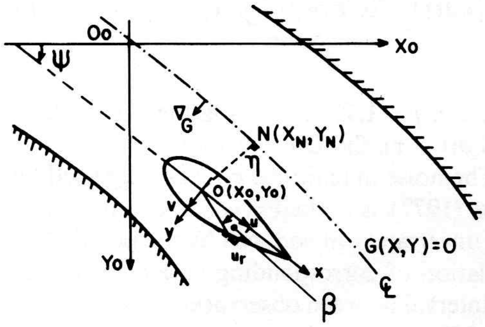

FIG. 2. Coordinate systems for ship motion.

hydrodynamic forces and moments depending upon the ship motion properties, rudder action and propeller conditions of operation; the hydrodynamic forces and moment F_{cx}, F_{cy} and M_c represent the disturbance due to the contribution of nonzero constant velocity and homogeneous current which are artificially separated from the hydrodynamic forces and moment, F_{hx}, F_{hy}, M_h, for the sake of modularity in the configuration of the simulation model (Leone, Sotelo and Cruz, 1973); F_w and M_w are the force and moment due to wind effects; m is the ship mass; u and v are respectively the O_x and O_y components of point O absolute velocity; r is the ship yaw angular velocity; x_G is the O_x center of gravity coordinate; I_z is the ship moment of inertia with respect to O_z; and the over dots indicate derivatives wih respect to time.

Combining equations (1)–(3) with the results of equations (A1)–(A9) of the Appendix, equations of the following type are obtained:

$$\Delta\dot{u} = f_1(\Delta u, v, r, \delta) + g_1(V_w, \psi, \psi_w), \tag{4}$$

$$\dot{v} = f_2(\Delta u, v, r, \eta(X_O, Y_O), \delta) + g_2(V_w, V_c, \psi, \psi_w, \psi_c), \tag{5}$$

$$\dot{r} = f_3(\Delta u, v, r, \eta(X_O, Y_O), \delta) + g_3(V_w, V_c, \psi, \psi_w, \psi_c), \tag{6}$$

where the $f_i(.)$ come from the contribution of F_{hx}, F_{hy}, M_h to equations (A1)–(A3); and the $g_i(.)$ come from the perturbations due to the effects of wind and current. Consider now that the components v_X and v_Y of point O velocity, in $O_O X_O Y_O$, are given by:

$$v_X = (u^* + \Delta u)\cos\psi - v\sin\psi. \tag{7}$$

$$v_Y = (u^* + \Delta u)\sin\psi + v\cos\psi. \tag{8}$$

Combining equations (7) and (8) with (4)–(6) and after taking the derivatives, one obtains

$$\dot{X}_O = v_X, \tag{9}$$

$$\dot{Y}_O = v_Y, \tag{10}$$

$$\dot{\psi} = r, \tag{11}$$

$$\dot{v}_X = -rv_Y + (f_1 + g_1)\cos\psi - (f_2 + g_2)\sin\psi, \tag{12}$$

$$\dot{v}_Y = rv_X + (f_1 + g_1)\sin\psi + (f_2 + g_2)\cos\psi, \tag{13}$$

$$\dot{r} = f_3 + g_3, \tag{14}$$

where the arguments of f_1 and g_1 are omitted for convenience of notation.

To complete the model, it is necessary to include the rudder angle dynamics. If V_R is the rudder rate and δ_c the commanded rudder angle, then an idealized model could be:

$$\dot{\delta} = \begin{cases} V_R, & \text{if } \delta_c - \delta > 0, \\ 0, & \text{if } \delta_c - \delta = 0, \\ -V_R, & \text{if } \delta_c - \delta < 0. \end{cases} \tag{15}$$

However, to have a model with the desired mathematical regularity and dynamics sufficiently close to that of equation (15), the following approximation was taken (Fig. 3):

$$\dot{\delta} = f_4(V_R, \Delta, \delta)$$
$$= \begin{cases} V_R(1 - \exp(-(\delta_c - \delta)/\Delta)), & \text{if } \delta_c - \delta \geqq 0, \\ -V_R(1 - \exp((\delta_c - \delta)/\Delta)), & \text{if } \delta_c - \delta < 0, \end{cases} \tag{16}$$

where Δ was adjusted to guarantee a response sufficiently close to the idealized model of equation (15).

3. STATE ESTIMATION

In the situation treated it is necessary to have a state estimator that performs well, in spite of errors in the modeled system dynamics. Only that part of the dynamics which is feasible to be modeled is explicitly considered in this state estimator. The unmodeled effects are taken care of by a dynamical model compensation technique (Tapley and Ingram, 1973) and by a heuristic procedure for evaluation of state noise level, as described below.

The lack of knowledge of the system model is treated by a scheme where the unmodeled

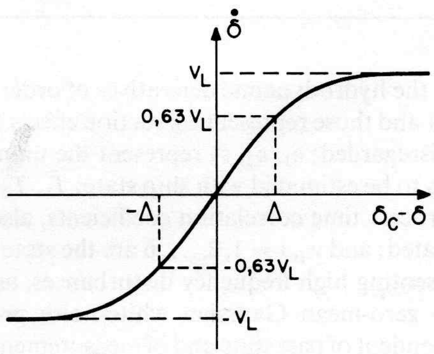

FIG. 3. Rudder motion model.

accelerations are estimated along with the original state variables. These accelerations are included as a first-order Gauss–Markov process, leading to a dynamical model of the system state estimator given by:

$$\dot{X}_0 = v_X,\ \dot{Y}_0 = v_Y, \quad (17)$$

$$\dot{v}_X = -rv_Y + \bar{f}_1(v_X, v_Y, \psi, r, \delta).\cos\psi - \bar{f}_2(v_X, v_Y, \psi, r, \delta).\sin\psi + \varepsilon_1 + w_1, \quad (18)$$

$$\dot{v}_Y = rv_X + \bar{f}_1(v_X, v_Y, \psi, r, \delta).\sin\psi + \bar{f}_2(v_X, v_Y, \psi, r, \delta).\cos\psi + \varepsilon_2 + w_2, \quad (19)$$

$$\dot{r} = \bar{f}_3(v_X, v_Y, \psi, r, \delta) + \varepsilon_3 + w_3, \quad (20)$$

$$\dot{\psi} = r, \dot{\delta} = f_4(V_R, \Delta, \delta), \quad (21)$$

$$\dot{\varepsilon}_1 = -\frac{1}{T_1}\varepsilon_1 + w_4, \dot{T}_1 = 0, \quad (22)$$

$$\dot{\varepsilon}_2 = -\frac{1}{T_2}\varepsilon_2 + w_5, \dot{T}_2 = 0, \quad (23)$$

$$\dot{\varepsilon}_3 = -\frac{1}{T_3}\varepsilon_3 + w_6, \dot{T}_3 = 0, \quad (24)$$

where $\bar{f}_i(.)$, $i = 1, 2, 3$, are coarse approximations, resulting from the $f_i(.)$ defined in equations (4)–(6),

$$\Delta\varepsilon_1(t_{k+1}, t_k) = \left(\left|\frac{\bar{F}_w}{X_{\dot{u}} - m}\right| + \left|\frac{(N_{\dot{r}} - I_z)(-\bar{F}_w + \bar{v}_c.Y_v)}{(Y_{\dot{v}} - m)(N_{\dot{r}} - I_z) - (N_{\dot{v}} - m.x_G).(Y_{\dot{r}} - m.x_G)}\right|\right).\bar{r}.\Delta t_k, \quad (28)$$

$$\Delta\varepsilon_2(t_{k+1}, t_k) = \Delta\varepsilon_1(t_{k+1}, t_k), \quad (29)$$

$$\Delta\varepsilon_3(t_{k+1}, t_k) = \left|\frac{(|\bar{F}_w.x_B| + |\bar{v}_c.N_v|).\bar{r}.(Y_{\dot{v}} - m)}{(Y_{\dot{r}} - m.x_G)(N_{\dot{v}} - m.x_G) - (N_{\dot{r}} - I_z)(Y_{\dot{v}} - m)}\right|.\Delta t_k, \quad (30)$$

$$\bar{F}_w(t_k) = C_R.\rho_a.(A_F + A_L)\bar{v}_w^2, \quad (31)$$

when the hydrodynamic derivatives of order higher than 1 and those representing suction effects (Y_η, N_η) are disregarded; $\varepsilon_1, \varepsilon_2, \varepsilon_3$ represent the unmodeled effects to be estimated with ship state; T_1, T_2, T_3 are the process time correlation coefficients, also to be estimated; and w_i, $i = 1, 2, \ldots, 6$ are the state noises, representing high frequency disturbances, assumed to be zero-mean Gaussian white-noise processes independent of past state and of measurement noise, with statistical characteristics given by:

$$E[w_i(t)] = 0,\ E[w_i(t)w_j(\tau)] = \sigma_i^2(t).\delta_{ij}.\delta(t - \tau), \quad (25)$$

where $i, j = 1, 2, \ldots, 6$; δ_{ij} is the Kronecker delta; and $\delta(t - \tau)$, the Dirac delta function.

The noise magnitude, $\sigma_i(t)$ as suggested by Gelb *et al.* (1977), is evaluated so as roughly to represent the uncertainty introduced by the possible range of variation of corresponding variables, $w_i(t)$, during an interval between observations. For t in $[t_k, t_{k+1}]$, the following heuristic approximations are taken:

$$\sigma_i^2(t) = \theta.\frac{P_{jj}(t_k/t_k)}{\Delta t_k}, \quad (26)$$

$$\sigma_{i+3}^2(t) = \frac{(\Delta\varepsilon_i(t_{k+1}, t_k))^2}{\Delta t_k}, \quad (27)$$

$i = 1, 2, 3$; $j = i + 2$; $0 < \theta \leq 1$; $\Delta t_k \triangleq t_{k+1} - t_k$, where θ is a parameter to be adjusted; $P_{jj}(t_k/t_k)$, $j = 3, 4, 5$, are the estimator error variances of v_X, v_Y and r; Δt_k is the time interval between observations; and $\Delta\varepsilon_i(t_{k+1}, t_k)$ are order of magnitude approximations for the possible range of variation of accelerations $\varepsilon_i(t)$ in $[t_k, t_{k+1}]$, taken as (Cruz, 1981):

where $\bar{r}$, $\bar{v}_c$, $\bar{v}_w$ are coarse but conservative guesses of ship maximum angular velocity, current velocity and wind velocity, respectively; and $\bar{F}_w$ an approximation of F_w (see Appendix).

The extended Kalman filter (Gelb *et al.*, 1977; Jazwinski, 1970) is then used to estimate the ship state vector, with system dynamics given by equations (19)–(24) and with noise corrupted observations taken at discrete-time instants. Adopting a concise notation to represent equations

(19)–(24), the problem can be posed as that of estimating the ship state vector $\mathbf{X}(t_k)$, with the equations:

$$\dot{\mathbf{X}} = F(\mathbf{X}, \mathbf{U}(t), t) + \mathbf{W}, \tag{32}$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{X}(t_k), t_k) + \mathbf{r}_k, \tag{33}$$

where $\mathbf{U}(t)$ is the control, given by δ_c in the rudder control problem; $\mathbf{y}_k$ is the observation vector at t_k, related to the state vector through the nonlinear relationship $\mathbf{h}_k(\mathbf{X}(t_k), t_k)$; $\mathbf{y}_k$ is corrupted by the Gaussian noise $\mathbf{r}_k$ which is independent of the state and whose statistics are given by:

$$E[\mathbf{r}_k] = 0,\ E[\mathbf{r}_k \mathbf{r}_k^T] = \mathbf{R}_k. \tag{34}$$

The estimates are then given by the corresponding sequential algorithm, which is summarized below:

$$\hat{\mathbf{X}}(t/t_k) = \hat{\mathbf{X}}(t_k/t_k) + \int_{t_k}^{t} \mathbf{F}(\hat{\mathbf{X}}(\tau/t_k), \delta_c, \tau)\, \mathrm{d}\tau, \tag{35}$$

$$\boldsymbol{\phi}(t, t_k) = \mathbf{I}_n + \int_{t_k}^{t} \mathbf{A}(\tau) . \boldsymbol{\phi}(\tau, t_k)\, \mathrm{d}\tau, \tag{36}$$

$$\mathbf{P}(t_{k+1}/t_k) = \boldsymbol{\phi}(t_{k+1}, t_k)\mathbf{P}(t_k/t_k)\boldsymbol{\phi}^T(t_{k+1}, t_k) + \mathbf{Q}_{k+1}, \tag{37}$$

$$\mathbf{Q}_{k+1} = \int_{t_k}^{t_{k+1}} \boldsymbol{\phi}(t_{k+1}, \tau)\mathbf{Q}(\tau)\boldsymbol{\phi}^T(t_{k+1}, \tau)\, \mathrm{d}\tau,$$

$$E[\mathbf{w}(t)\mathbf{w}^T(\tau)] = \mathbf{Q}(\tau) . \delta(t - \tau),$$

$$\mathbf{H}_{k+1} = \frac{\partial}{\partial \mathbf{X}} \mathbf{h}_{k+1}(\hat{\mathbf{X}}(t_{k+1}/t_k), t_{k+1}),$$

$$\mathbf{A}(\tau) = \frac{\partial \mathbf{F}}{\partial \mathbf{X}}(\hat{\mathbf{X}}(\tau/t_k), \delta_c, \tau),$$

$$\mathbf{K}_{k+1} = \mathbf{P}(t_{k+1}/t_k)\mathbf{H}_{k+1}^T(\mathbf{H}_{k+1}\mathbf{P}(t_{k+1}/t_k)\mathbf{H}_{k+1}^T + \mathbf{R}_{k+1})^{-1}, \tag{38}$$

$$\hat{\mathbf{X}}(t_{k+1}/t_k) = \hat{\mathbf{X}}(t_{k+1}/t_k) + \mathbf{K}_{k+1}(\mathbf{y}_{k+1} - h_{k+1}(\hat{\mathbf{X}}(t_{k+1}/t_k), t_{k+1})), \tag{39}$$

$$\mathbf{P}(t_{k+1}/t_{k+1}) = (\mathbf{I}_n - \mathbf{K}_{k+1}\mathbf{H}_{k+1})P(t_{k+1}/t_k), \tag{40}$$

where $\mathbf{I}_n$ is the identity matrix of order n.

4. CONTROL LAW

The problem considered here is of the type where the guidance information is given in the form of a nonlinear geometric relationship, corresponding to a curve to be followed. There is neither a reference state trajectory nor any explicit time schedule, to guide the control action. Thus, the usual scheme of linear perturbation about a reference solution and the approach of obtaining the control increment law as solution of a LQG problem (Bryson and Ho, 1975; Gelb *et al.*, 1977) cannot be applied. In the procedure proposed, an optimum criterion of minimum deviation from the guidance curve is adopted (Rios-Neto and Cruz, 1979). Based on the information given by the state estimate, the control correction is constrained to drive the state closer to the prescribed path, and with favorable kinematic conditions to follow it, after one time interval of prediction. In a typical interval $[t_k, t_{k+1}]$, using the state estimate $\hat{\mathbf{X}}(t_k/t_k)$ and the state error covariance matrix $\mathbf{P}(t_k/t_k)$, the objective is to get a stepwise feedback control correction, $\delta\mathbf{U}_k$, so as to reach a system state vector at t_{k+1}, satisfying as close as possible the guidance condition in the following manner: the control,

$$\mathbf{U}(t) = \bar{\mathbf{U}}_k + \delta\mathbf{U}_k,\ t_k \leqq t \leqq t_{k+1}, \tag{41}$$

is used in the equation (32), resulting in:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\bar{\mathbf{X}} + \delta\mathbf{X}) = \mathbf{F}(\bar{\mathbf{X}} + \delta\mathbf{X}, \bar{\mathbf{U}}_k + \delta\mathbf{U}_k, t) + \mathbf{W}, \tag{42}$$

where

$$\dot{\bar{\mathbf{X}}} = \mathbf{F}(\bar{\mathbf{X}}, \bar{\mathbf{U}}_k, t),\ \bar{\mathbf{X}}(t_k) = \hat{\mathbf{X}}(t_k/t_k), \tag{43}$$

$$E[\delta\mathbf{X}(t_k)\delta\mathbf{X}^T(t_k)] = \mathbf{P}(t_k/t_k), \tag{44}$$

so as to obtain a $\delta\mathbf{X}(t_{k+1})$, to satisfy the guidance condition at t_{k+1}, i.e.:

$$\mathbf{s}(\bar{\mathbf{X}}(t_{k+1}) + \delta\mathbf{X}(t_{k+1})) = 0, \tag{45}$$

where $\mathbf{s}(\mathbf{X}(t_{k+1}))$ is a vector relationship representing the guidance condition of getting closer to the prescribed path and with favorable kinematic conditions to follow it. The control $\bar{\mathbf{U}}_k$ is the value previously calculated and applied in the interval $[t_{k-1}, t_k]$. Taking the first-order variational equations associated to equation (42), one obtains:

$$\delta\mathbf{X}(t_{k+1}) = \boldsymbol{\phi}(t_{k+1}, t_k)\delta\mathbf{X}(t_k) + \boldsymbol{\Gamma}(t_{k+1}, t_k)\delta\mathbf{U}_k + \mathbf{W}_{k+1}, \tag{46}$$

$$\mathbf{W}_{k+1} \sim N(0, \mathbf{Q}_{k+1}), \boldsymbol{\Gamma}(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} \boldsymbol{\phi}(t_{k+1}, \tau) . \frac{\partial}{\partial \mathbf{U}} \mathbf{F}(\bar{\mathbf{X}}(\tau), \bar{\mathbf{U}}(\tau), \tau)\, \mathrm{d}\tau,$$

where $\boldsymbol{\phi}(t_{k+1}, t_k)$ and $\mathbf{Q}_{k+1}$ are as defined in Section 3, in the equations (36) and (37), respectively. Combining this with a Taylor series expansion of equation (45), about the over-bar nominal values, one gets:

$$\mathbf{Z}_{k+1} = \mathbf{S}_{k+1}\mathbf{U}_k + \mathbf{v}_{k+1}, \tag{47}$$

where:

$$\mathbf{S}^*_{k+1} = \frac{d}{d\mathbf{X}}\mathbf{s}(\bar{\mathbf{X}}(t_{k+1})); \; \mathbf{S}_{k+1} = \mathbf{S}^*_{k+1}.\mathbf{\Gamma}(t_{k+1}, t_k);$$

$$\mathbf{Z}_{k+1} = \mathbf{S}_{k+1}\bar{\mathbf{U}}_k - \mathbf{s}(\bar{\mathbf{X}}(t_{k+1}));$$

$$\mathbf{v}_{k+1} = \mathbf{S}^*_{k+1}.(\mathbf{\phi}(t_{k+1}, t_k).\delta\mathbf{X}(t_k) + \mathbf{W}_{k+1}) + \mathbf{\varepsilon}_{k+1}. \quad (48)$$

$\mathbf{\varepsilon}_{k+1}$ in equation (48) represents the linearization errors, modeled by a Gaussian zero mean random sequence, with a dispersion compatible with the order of magnitude of the truncated higher-order terms and given by (Cruz, 1981):

$$/\mathbf{\varepsilon}_{k+1}/ \leqq /\mathbf{S}^*_{k+1}.\delta\mathbf{X}(t_{k+1})/ \leqq /\mathbf{S}^*_{k+1}/ .(/\mathbf{\phi}(t_{k+1}, t_k)/./\delta\mathbf{X}(t_k)/ + /\mathbf{\Gamma}(t_{k+1}, t_k)/ ./\delta\mathbf{U}_k/ + /\mathbf{W}_{k+1}/), \quad (49)$$

where, for a given matrix A, $/A/ \triangleq (|a_{ij}|)$, and the inequality is meant for the corresponding absolute values of elements in each member. Thus, from equations (48) and (49) one obtains:

$$/\mathbf{v}_{k+1}/ \leqq \mathbf{S}^*_{k+1}/.(/\mathbf{\phi}(t_{k+1}, t_k)/./2\delta\mathbf{X}(t_k)/ + /\mathbf{\Gamma}(t_{k+1}, t_k)/./\delta\mathbf{U}_k/ + /2\mathbf{W}_{k+1}/). \quad (50)$$

Since the objective is to establish the order of magnitude of the error dispersion, a good approximation seems to be to take $/\delta\mathbf{X}(t_k)/$ and $/W_{k+1}/$ as vectors whose elements are the square root of the diagonal elements of $\mathbf{P}(t_k, t_k)$ and $\mathbf{Q}_{k+1}$, respectively; and $/\delta\mathbf{U}_k/$ as a vector with the elements calculated as the maximum admissible variations of the corresponding control components, in a typical interval of discretization. With these choices, a vector $\mathbf{e}_{k+1}$ is defined by:

$$\mathbf{e}_{k+1} \triangleq /\mathbf{S}^*_{k+1}/.(/\mathbf{\phi}(t_{k+1}, t_k)/./2\delta\mathbf{X}(t_k)/ + /\mathbf{\Gamma}(t_{k+1}, t_k)/./\delta\mathbf{U}_k/ + /2\mathbf{W}_{k+1}/) \quad (51)$$

and the covariance matrix of $\mathbf{v}_{k+1}$ is taken as:

$$\mathbf{V}_{k+1} = \mathrm{diag}(\alpha_i e^2_{k+1,i}, i = 1, 2, \ldots, m), \quad 0 < \alpha_i \leqq 1, \quad (52)$$

where $e_{k+1,i}$ is the ith component of $\mathbf{e}_{k+1}$; and α_i are adjustable parameters which allow one to calibrate the relative weight to be given to the ith control condition of equation (47).

With the stochastic modeling given to $\mathbf{v}_{k+1}$, the control determination problem is reduced to one of linear optimal parameter estimation with no *a priori* information. Thus, the best estimate of $\mathbf{U}_k$ is $\hat{\mathbf{U}}_k$, given by a least-squares fit:

$$\hat{\mathbf{U}}_k = (\mathbf{S}^T_{k+1}.\mathbf{V}^{-1}_{k+1}.\mathbf{S}_{k+1})^{-1}.\mathbf{S}^T_{k+1}.\mathbf{V}^{-1}_{k+1}.\mathbf{Z}_{k+1}. \quad (53)$$

5. TEST AND RESULTS

5.1. *Simulation and control guidance conditions*

Tests were carried out by digital simulation using a PDP-15/77-D.O.S. with principal memory of 32k words. A merchant Mariner class vessel (Chislett and Strøm-Tejsen, 1965) steering in restricted waters was considered. The steering situation was that of conducting the ship along the center line of a channel, under the perturbations of wind, current and suction effects. To simulate the real system response to the inputs of the automatic pilot controller (Fig. 1), the model defined by the equations (9)–(16), in Section 2, was used. The hydrodynamic derivatives up to third order were as given in Chislett and Strøm-Tejsen (1965) for a ship with the following main characteristics:

Length: 160.1 m; breadth: 22 m;
Draft: 7.5 m; displacement: 16 600 ton;
Velocity (u^*): 15 knots; max. rudder angle (δ_{mx}): 35°;
Rudder rate (V_R): 4°/s; projected frontal area (A_F): 100 m^2;
Projected lateral area (A_L): 800 m^2;
A_L center of gravity coord. (x_L): 2 m.

The disturbing effects of wind and current were varied according to the test cases and the effects of suction were characterized through the dimensionless (with respect to water density, ship length and ship velocity variables) hydrodynamic derivatives:

$$Y_\eta = 0.235\mathrm{E}-02, \; N_\eta = -0.310\mathrm{E}-03.$$

The simulated real system response was used to generate observations of range and range-rate to some fixed land points together with those of heading angle (ψ), yaw-rate (η) and rudder angle (δ). These simulated measurements were then corrupted by pseudorandom Gaussian noise, representative of instrument errors, with standard deviation as follows:

Range: 3.0 m, range-rate: 0.05 m/s;
Yaw-rate: 0.2°/s; heading angle: 0.1°;
Rudder angle: 0.5°.

The control guidance conditions of equation (45) were taken to accomplish the objective of steering the ship along the channel center line (Fig. 2), with the rudder as the control element. The first condition was to constrain a ship reference point B of Ox to be over the channel center line:

$$s_1(X_0, Y_0, x_B, \psi) = \eta_B = 0, \quad (54)$$

where η_B is the distance from point B to the center line. The second was to constrain the ship to maneuver keeping head direction parallel to channel center line, yielding:

$$s_2(X_0, Y_0, \psi) = \cos\psi \cdot \frac{\partial}{\partial X_0} G(X_0, Y_0) + \sin\psi \cdot \frac{\partial}{\partial Y_0} G(X_0, Y_0) = 0, \quad (55)$$

where $G(X, Y) = 0$ is the channel center line equation. The two previous conditions could be called configuration conditions, since they involve only the position variables. To guarantee the tendency of keeping over the channel center line, two kinematical conditions were also taken. The first imposes the ship velocity vector to be parallel to the prescribed path:

$$s_3(V_X, V_Y, X_0, Y_0) = v_X \cdot \frac{\partial}{\partial X_0} G(X_0, Y_0) + v_Y \cdot \frac{\partial}{\partial Y_0} G(X_0, Y_0) = 0. \quad (56)$$

The second imposes the ship maneuvering to maintain parallelism with the prescribed path:

$$s_4(v_X, v_Y, r, X_0, Y_0) = r - \frac{u_r}{\rho} = 0, \quad (57)$$

where u_r is the magnitude of ship velocity (Fig. 2) and ρ, the local radius of curvature of channel center line.

Note that the conditions of equations (55) and (56) cannot be met simultaneously, due to the fact that the drift angle β (Fig. 2) becomes different from zero as the ship maneuvers. However, since they are to be only approximately satisfied, and in view of their distinct motivations, they were both included. Note also that the condition of equation (56) should refer to point B instead of O; however, point O was used under the assumption that ship rotation is small when compared to translational motion.

5.2. *Test conditions*

For the application of ship steering along the center line of a channel, many adverse situations in terms of (i) the lack of knowledge of the dynamical model; (ii) the complexity of the geometry of the path to be followed; and (iii) the environmental disturbances were considered when the preliminary digital simulation testing was carried out. The results obtained were all of the same level of quality as those of the test case selected for presentation. This case corresponds to a combination of critical conditions involving the adverse situations mentioned.

Steering and observation conditions. A channel with sinusoidal form was considered. The radius of curvature in sinusoidal peaks is 405 m, whereas the ship tactical diameters are 794 and 829 m for the rudder at 35° starboard and port, respectively. The wind speed was taken as 40 knots, in the direction opposite to O_0Y_0, and the current speed as 2 knots, locally parallel to channel center line, in the same direction as the ship motion.

Initial conditions of the motion of the real system are given in Table 1.

To simulate range and range-rate, point R ($x_R = 100$ m) in the ship was taken as the reference for measurements to four landmarks, P_I, distributed along channel (Fig. 4) margins as given also in Table 1.

State estimator and controller. Discretization time intervals of 2 s and 8 s were used for state estimator and controller, respectively. Dynamic model of the automatic pilot was that of equations (17)–(24). The initial conditions were taken as those of Table 2.

The first-order hydrodynamic derivatives used were those of the real system corrupted by the percentage errors as given in Table 3. These errors were produced as outcomes of a numerically

TABLE 1. SIMULATION CONDITIONS

Real System Initial Conditions			
$X_0(t_0) = Y_0(t_0) = 0.0$, $\psi(t_0) = 60^\circ$			
$v_X(t_0) = 7.5$ knots, $v_Y(t_0) = 13.0$ knots			
$r(t_0) = 0.0^\circ$/s, $\delta(t_0) = 0.0^\circ$			
Landmarks Along Channel Margin			
X(1) = 1000 m	X(2) = 1000 m	X(3) = 3000 m	X(4) = 3000 m
Y(1) = 500 m	Y(2) = 1500 m	Y(3) = -500 m	Y(4) = 1500 m

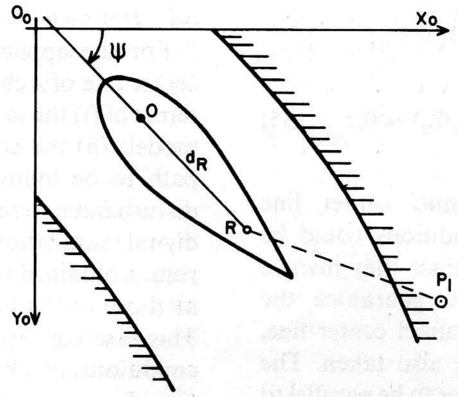

FIG. 4. Range measurement.

TABLE 2. STATE ESTIMATOR AND CONTROLLER INITIAL CONDITIONS

Variable	Initial Value	Standard Deviation
X_0	-10.9 m	5.0 m
Y_0	43.5 m	5.0 m
ψ	0.4^o	2.0^o
v_X	15.2 knots	0.1 knots
v_Y	0.0	0.1 knots
r	0.1^o/s	0.1^o/s
δ	0.0^o	0.5^o
ε_1	0.029 m/s²	0.033 m/s²
ε_2	-0.035 m/s²	0.033 m/s²
ε_3	$-0.05^o/s^2$	$0.025^o/s^2$
T_1	1000.0 s	200.0 s
T_2	1000.0 s	200.0 s
T_3	1000.0 s	200.0 s

TABLE 3. ERRORS IN HYDRODYNAMIC DERIVATIVES

X_u: -25.5%	$X_{\dot u}$: 20.9%	N_v: -19.6%	$Y_{\dot r}$: -0.3%
Y_v: 5.0%	$N_{\dot r}$: -6.0%	N_r: 6.0%	Y_r: 19.2%
N_δ: 10.1%	Y_δ: -14.6%	$Y_{\dot v}$: -14.6%	$N_{\dot v}$: 13.8%
N^*_u: 8.6%	Y^*_u: 1.3%	N^*: 5.2%	Y^*: 17.1%

simulated normal distribution of zero mean and standard deviation of 15% of the real values. They represent experimental errors present in the determination of hydrodynamic derivatives.

The values adopted to characterize state noise magnitude, in equations (28)–(31), and those to characterize the calibration parameters in equations (16), (51), (52) and (53) were as in Table 4.

5.3. *Results*

As shown in Fig. 5, even under severely restricted conditions, the state estimation and control law procedures lead to control actions that steer the ship very closely to the channel center line and the largest deviations, near curvature peaks, are of the order of magnitude of twice the ship breadth (50 m). Figures 7–9 show the ability of the dynamical model compensation technique to estimate the unmodeled accelerations. Note that there are occasions when the unmodeled acceleration has magnitude close to the total acceleration, as is the case around 200 and 600 s, for the O_0X_0 component. Note also that near those points the unmodeled part of the angular acceleration (Fig. 9) is much greater than the total acceleration, which is close to zero. This implies that modeled and unmodeled parts have the same order of magnitude, but with opposite signs. Finally, from Figs. 10–13 it can be seen that the extended Kalman filter gives standard deviations of the state estimation errors which are conservative evaluations of real estimation errors in position (Δ_{XY} in Fig. 10 and Δ_ψ in Fig. 11) and velocity (Δ_V in Fig. 12 and Δ_r in Fig. 13). The maximum values for these estimation errors after filter convergence, were:

$$\Delta_{XY} = 0.22\,\text{m}, \qquad \Delta_\psi = 0.12°,$$

$$\Delta_V = 0.12\ \text{knots}, \qquad \Delta_r = 0.045°/\text{s}.$$

TABLE 4. ESTIMATOR AND CONTROLLER PARAMETERS

State Noise Parameters				
$\Theta = 0.05$	$\bar{r} = 1.5°/s$	$\bar{v}_c = 3$ m/s	$\bar{v}_w = 30$ m/s	
Calibration Parameters				
$\Delta = 0.1$	$\alpha_1 = 1.0$	$\alpha_2 = 1.0$	$\alpha_3 = 1.0$	$\alpha_4 = 0.3$
$/\delta \underline{U}_k/ = /\delta(\delta_c)/ = 30°$, $x_B = 8.0$ m				

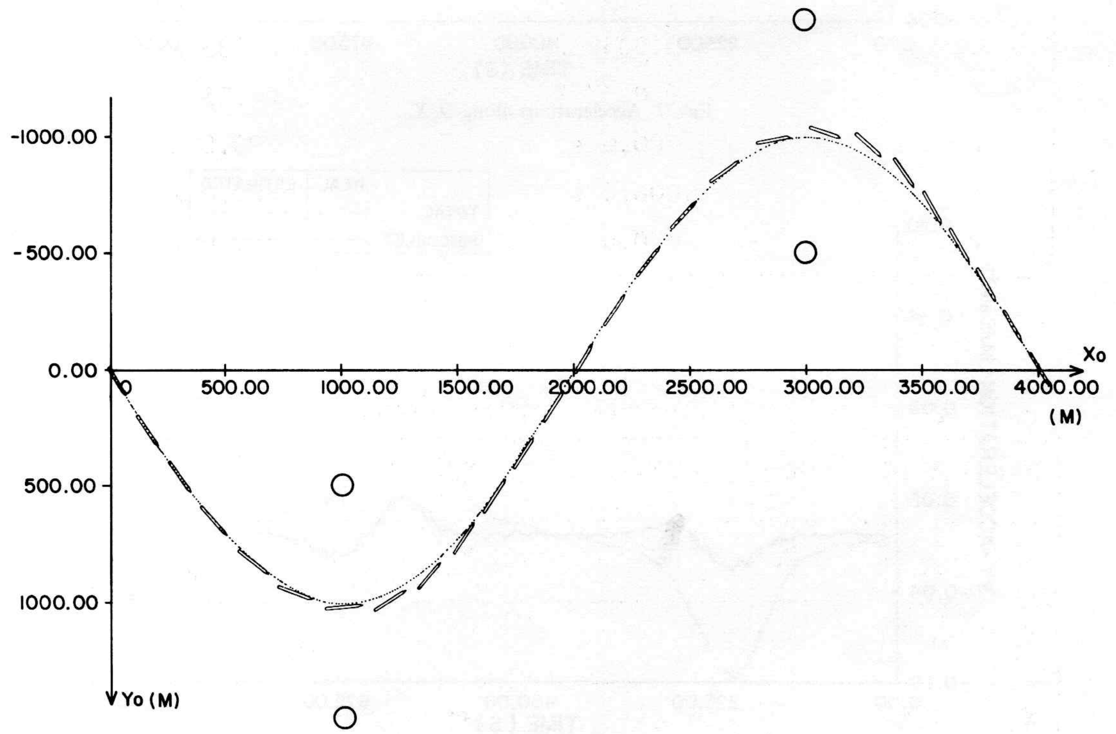

FIG. 5. Channel center line and ship controlled path.

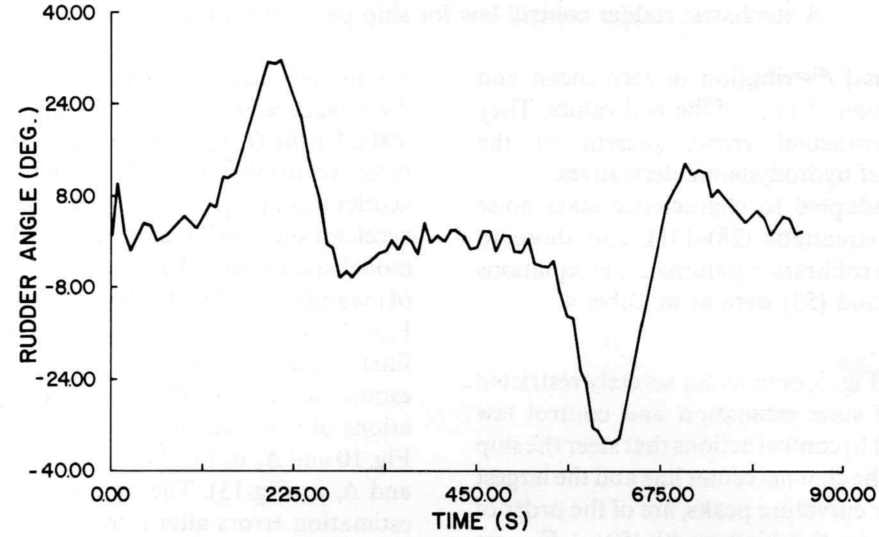

FIG. 6. Rudder angle time history.

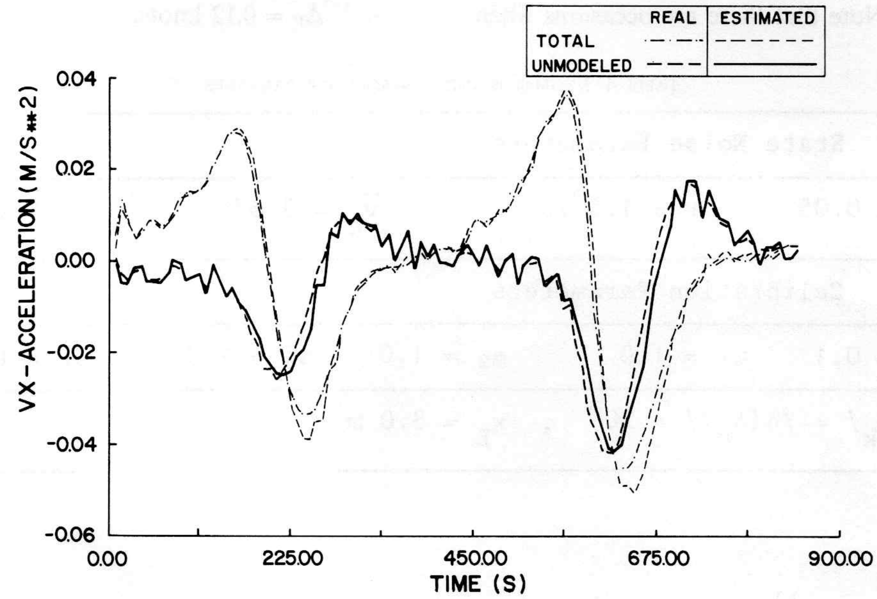

FIG. 7. Accelerations along O_0X_0.

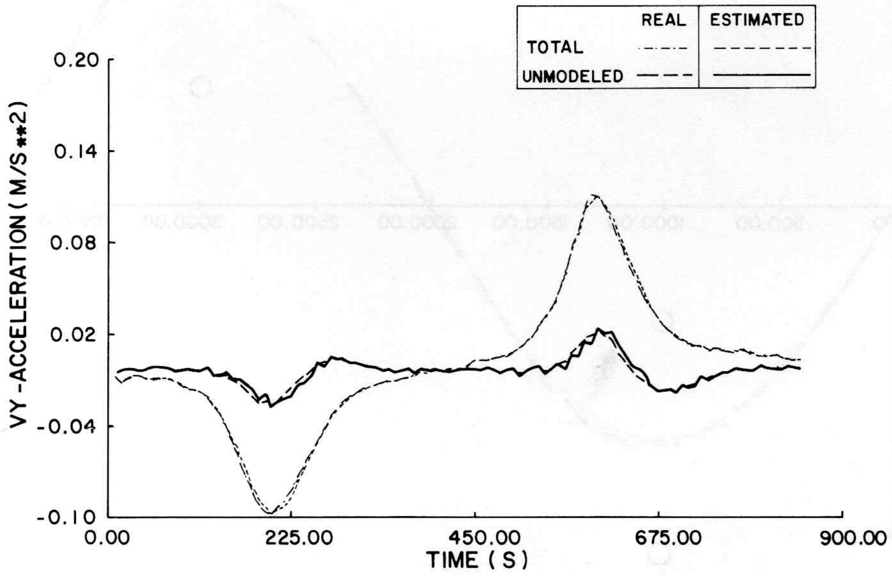

FIG. 8. Accelerations along O_0Y_0.

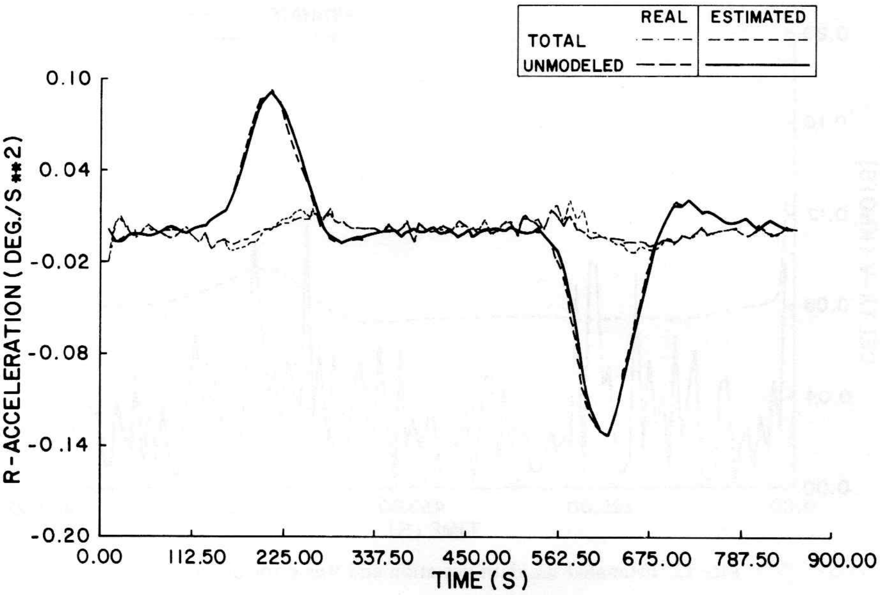

FIG. 9. Angular accelerations.

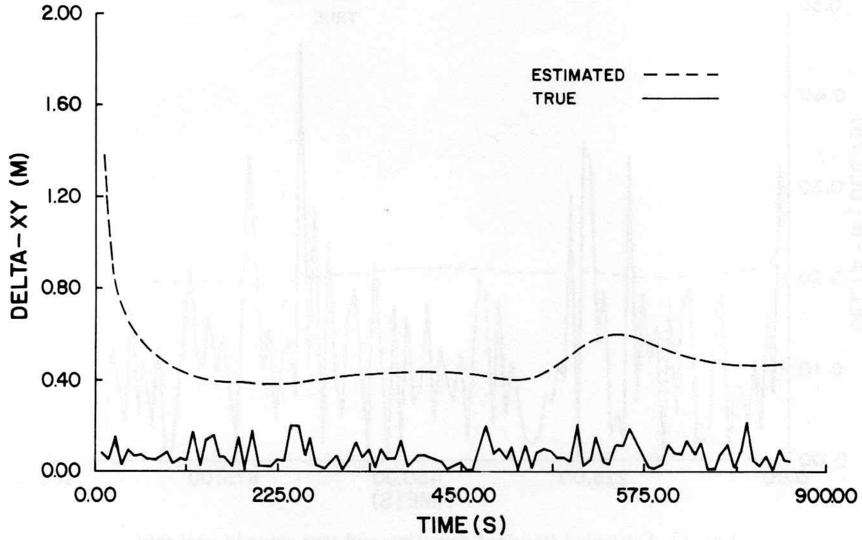

FIG. 10. Estimated standard deviation and true error in position.

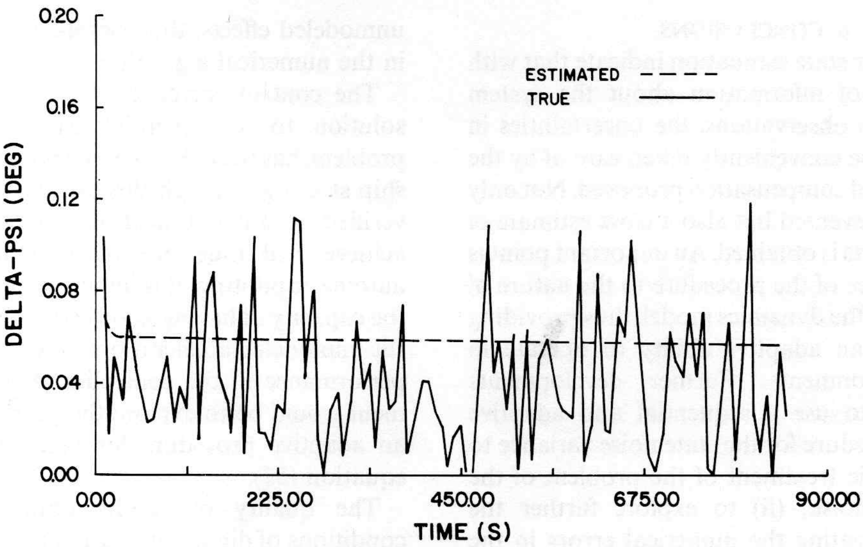

FIG. 11. Estimated standard deviation and true error in heading angle.

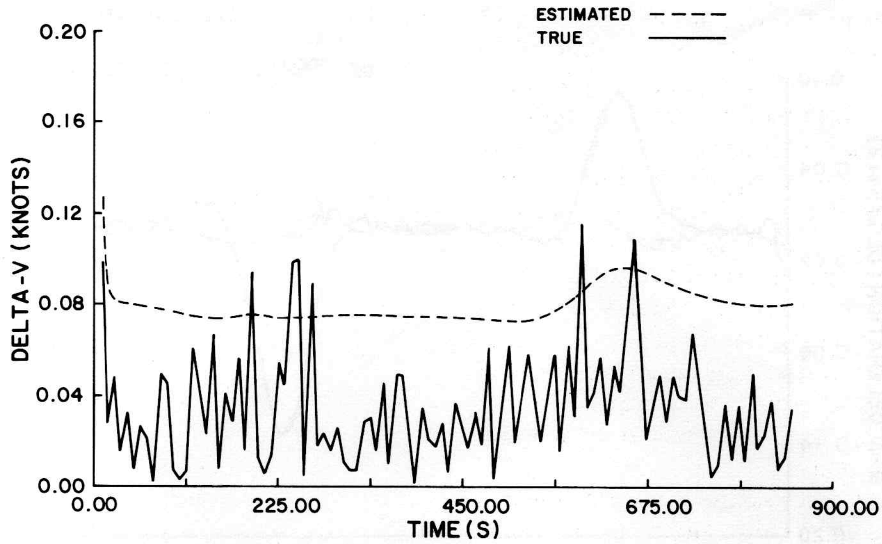

FIG. 12. Estimated standard deviation and true error in velocity.

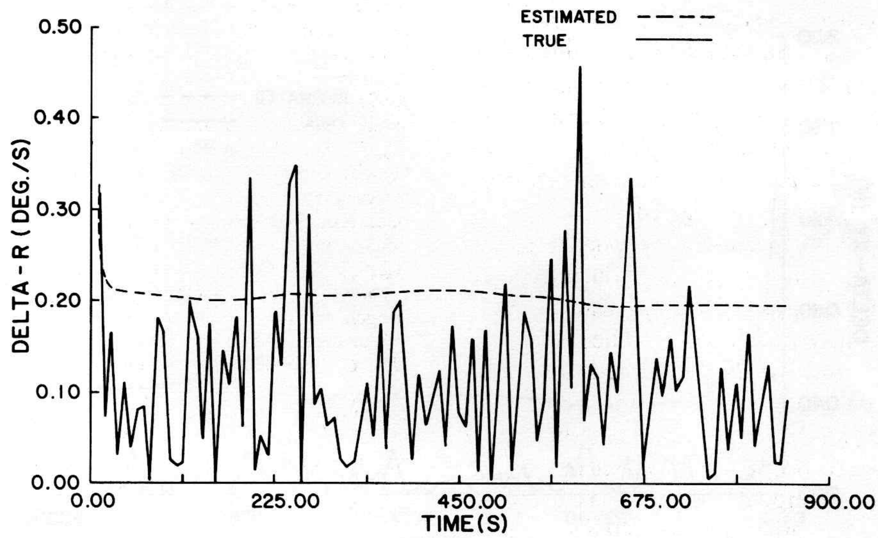

FIG. 13. Estimated standard deviation and true error in yaw-rate.

6. CONCLUSIONS

The results for state estimation indicate that with the high level of information about the system provided by the observations, the uncertainties in the model can be conveniently taken care of by the dynamical model compensation proposed. Not only is divergence prevented but also a close estimate of unmodeled effects is obtained. An important point is the independence of the procedure to the nature of uncertainties in the dynamics model, thus providing the pilot with an adaptive ability to operate in different environments. Further developments would be: (i) to use a sequential and adaptive estimation procedure for the state noise variance to give a systematic treatment of the problem of the level of state noise; (ii) to explore further the possibility of treating the numerical errors in the prediction phase of the extended Kalman filter as unmodeled effects, thus simplifying the conditions in the numerical algorithms.

The control correction law, formulated as the solution to a sequential parameter estimation problem, has been shown to be potentially useful for ship steering. Though this aspect has not yet been verified, results obtained indicate its suitability to achieve real time operation as in the case of automatic piloting. It is important to point out that the capacity of having good estimates of the state of the unmodeled accelerations was essential for good performance of the controller. A further development would be to explore the possibility of having an adaptive procedure for determining the α_i in equation (52).

The quality of results obtained under the conditions of digital simulation reported in Section 5 constitutes only an indication of feasibility to employ the proposed procedures in the automatic steering of ships in channels. Before final qualification, one should investigate (i) the effects of other types of perturbations—such as those of waves and of shallow waters; (ii) the results of simulation tests for other types of ships, for example supertankers exhibiting an unstable dynamical behavior; (iii) the sensitivity of the proposed procedure to extreme situations, for example, low-speed operation; (iv) also the quality of results under full-scale tests. However, good performance in other situations can be expected keeping in mind the inherent characteristic of the state estimator to deal with state noise and its capacity of adaptive model compensation.

At first glance the procedure might be considered too complicated for practical implementation. One could argue that there are too many equations to be solved and too many parameters to be calibrated [as the α_i in equation (52)]. However, with present computer hardware and software resources, it should be feasible to implement the autopilot in a ship on-board minicomputer. Regarding the calibration of parameters, they can be determined in advance in the development phase, through simulation tests. The tests made have shown that this task can be easily accomplished.

As a final comment, one should emphasize some of the characteristics that are typical of the proposed procedures. Depending upon the application, they can constitute advantages over other existing solutions. They are: (i) the extended adaptive versions of the estimator and of the controller permit the use of nonlinear models, thus not imposing any constraint on the structure or complexity of these models; (ii) and, as a direct consequence, the potential capacity of the autopilot executing any physically feasible maneuver, as long as the maneuver is characterized by a well-defined geometrical path to be followed.

APPENDIX: SIMULATION MODEL OF MOTION

For the purposes of this study it was assumed that it would be satisfactory to consider the following functional relationships for hydrodynamic forces and moments:

$$F_{hx} + F_{cx} = X(u - u_c, v - v_c, \dot{u}, \dot{v}, r, \dot{r}, \eta, \delta),$$
$$F_{hy} + F_{cy} = Y(u - u_c, v - v_c, \dot{u}, \dot{v}, r, \dot{r}, \eta, \delta),$$
$$M_h + M_c = N(u - u_c, v - v_c, \dot{u}, \dot{v}, r, \dot{r}, \eta, \delta),$$

where u_c and v_c are the surge and sway components of the absolute constant velocity V_c of current; η is the distance from point O to the channel center line; δ is the rudder deflection; and the other variables as defined in equations (1)–(3). If a Taylor series expansion is taken about the nominal conditions corresponding to zero velocity current and to straight-ahead motion at constant speed ($u^* =$ constant, $v^* = 0$), with rudder amidships and no deviation from channel center line, the following nonlinear model results as an approximation to equations (1)–(3):

$$\begin{aligned}(m - X_{\dot{u}})\dot{u} = {} & X_u\Delta u + \tfrac{1}{2}(X_{uu}\Delta u^2 + X_{vv}v^2 + (X_{rr} + 2mx_G)r^2 \\ & + X_{\delta\delta}\delta^2) + (X_{vr} + m)vr + X_{v\delta}v\delta + X_{r\delta}r\delta \\ & + \tfrac{1}{2}(X_{vvu}v^2\Delta u + X_{rru}r^2\Delta u + X_{\delta\delta u}\delta^2\Delta u) \\ & + X_{vru}vr\Delta u + X_{v\delta u}v\delta\Delta u + X_{r\delta u}r\delta\Delta u + F_{wx} + F_{cx},\end{aligned} \quad \text{(A1)}$$

$$\begin{aligned}(m - Y_{\dot{v}})\dot{v} + (mx_G - Y_{\dot{r}})\dot{r} = {} & Y^* + Y_u^*\Delta u + Y_v v + (Y_r - mu)r \\ & + Y_\eta\eta + Y_\delta\delta + Y_{uu}^*\Delta u^2 + Y_{vu}v\Delta u + Y_{ru}r\Delta u \\ & + Y_{\delta u}\delta\Delta u + Y_{vr\delta}v_{r\delta} + \tfrac{1}{2}(Y_{vuu}v\Delta u^2 + Y_{vrr}vr^2 \\ & + Y_{v\delta\delta}v\delta^2 + Y_{ruu}r\Delta u^2 + Y_{rvv}rv^2 + Y_{r\delta\delta}r\delta^2 \\ & + Y_{\delta uu}\delta\Delta u^2 + Y_{\delta vv}\delta v^2 + Y_{\delta rr}\delta r^2) \\ & + \tfrac{1}{6}(Y_{vvv}v^3 + Y_{rrr}r^3 + Y_{\delta\delta\delta}\delta^3) + F_{wy} + F_{cy},\end{aligned} \quad \text{(A2)}$$

$$\begin{aligned}(mx_G - N_{\dot{v}})\dot{v} + (I_z - N_{\dot{r}})\dot{r} = {} & N^* + N_u^*\Delta u + N_v v \\ & + (N_r - mx_G u)r + N_\eta\eta + N_\delta\delta + N_{uu}^*\Delta u^2 \\ & + N_{vu}v\Delta u + N_{ru}r\Delta u + N_{\delta u}\delta\Delta u + N_{vr\delta}vr\delta \\ & + \tfrac{1}{2}(N_{vuu}v\Delta u^2 + N_{vrr}vr^2 + N_{v\delta\delta}v\delta^2 + N_{ruu}r\Delta u^2 \\ & + N_{rvv}rv^2 + N_{r\delta\delta}r\delta^2 + N_{\delta uu}\delta\Delta u^2 + N_{\delta vv}\delta v^2 \\ & + N_{\delta rr}\delta r^2) + \tfrac{1}{6}(N_{vvv}v^3 + N_{rrr}r^3 + N_{\delta\delta\delta}\delta^3) \\ & + M_w + M_c,\end{aligned} \quad \text{(A3)}$$

where Δu is the deviation of u from u^*; v is the deviation from the zero value of v^*. The usual notation has been used to indicate the hydrodynamic derivatives corresponding to the partial derivatives in the Taylor series expansion (Cruz, 1981; Abkowitz, 1969; Strøm-Tejsen, 1965). Only nonzero and nonnegligibly small terms were kept and the hydrodynamic forces and moment, F_{cx}, F_{cy} and M_c, represent the disturbance of nonzero constant velocity current, artificially separated from the hydrodynamic forces and moment, F_{hx}, F_{hy} and M_h, which occur in the absence of current (Leone, Sotelo and Cruz, 1973). As a consequence of body symmetry, the hydrodynamic derivatives Y_u, Y_{uu}, Y_{uuu}, $Y_{\dot{u}}$, N_u, N_{uu}, N_{uuu} and $N_{\dot{u}}$ are all zero. The zero order terms Y^* and N^* account for nonsymmetrical side forces (for instance, due to propeller action). To assure that the changes with speed in these side forces are also taken into account, the terms Y_u^*, Y_{uu}^*, N_u^* and N_{uu}^* are included (Strøm-Tejsen, 1965). For currents with absolute velocity V_c (with respect to $O_0X_0Y_0$) in the direction of ψ_c (same orientation as ψ, ψ_w), the resulting force in the direction of Oy (F_{cx}) was disregarded and the corresponding moment M_c were approximately modeled by (Leone, Sotelo and Cruz, 1973):

$$F_c = F_{cy} = (V_c \sin(\psi - \psi_c)) \,.\, Y_v + \tfrac{1}{6}(V_c \sin(\psi - \psi_c))^3 \,.\, Y_{vvv}, \quad \text{(A4)}$$

$$M_c = (V_c \sin(\psi - \psi_c)) \,.\, N_v + \tfrac{1}{6}(V_c \sin(\psi - \psi_c))^3 \,.\, N_{vvv}. \quad \text{(A5)}$$

For winds with absolute velocity V_w (with respect to $O_0X_0Y_0$) in the direction of ψ_w (measured in the same orientation as ψ in Fig. 2), the resulting force F_w (applied on point $L \in Ox$ with coordinate x_L, in the body fixed $Oxyz$ system) and the corresponding moment M_w were approximately modeled by (Leone, Sotelo and Cruz, 1973):

$$F_w = C_R \,.\, \rho_a \,.\, (A_F \cos^2(\psi_w - \psi) + A_L \sin^2(\psi_w - \psi) \,.\, V_w^2, \quad \text{(A6)}$$

resulting:

$$F_{wx} = -F_w \,.\, \cos(\psi_w - \psi), \quad \text{(A7)}$$

$$F_{wy} = -F_w \,.\, \sin(\psi_w - \psi), \quad \text{(A8)}$$

$$M_w = F_{wy} \,.\, x_L, \quad \text{(A9)}$$

where A_L and A_F are the projected lateral and frontal ship areas above the line of water; C_R is the air resistance coefficient; and ρ_a is the air density. The coarse approximation of taking wind absolute velocity was considered reasonable since the main purpose was to represent only the more significant disturbance of lateral winds.

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