FUNCTIONAL LINK NEURAL NETWORKS IN AERODYNAMIC MODELING OF AIRCRAFT

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Abstract. An artificial neural network (ANN) of the functional link network (FLN) type is applied in the problem of aerodynamic modeling of an aircraft. In the proposed method, a functional link network is used to identify the stability and control derivatives as they appear in the equations of motion of the aircraft. State and control variables of the aircraft are used to train the network. Exploring the structure of the FLN, the training is done with a stochastic linear parameter estimation algorithm where training data accuracy is considered. To arrive at neural networks that model accurately but are parsimonious, a pruning technique known as the Optimal Brain Surgeon (OBS) is used in order to eliminate the least significant neural connections. To demonstrate the method tests are conducted using simulated data to identify a nonlinear longitudinal dynamic model of a high performance aircraft.

Keywords: *Aerodynamic Coefficients, Aerodynamic Modeling, Functional Link Network*

1. Introduction

Though significant advances have been made in the estimation of aircraft model parameters (Greenberg, 1951; Klein, 1981; Maine and Iliff, 1985; Klein, 1989; Jategaonkar and Plaestschke, 1989; Jategaonkar and Thielecke, 1994, 2000; Hamel and Jategaonkar, 1996; Bauer and Andrisani, 1990; Curvo 2000), there is much to be explored. The use of Artificial Neural Networks (ANN) as universal approximators (e.g., Haykin, 1994) opens new possibilities yet to be exploited, especially in the development of on board real or near real time estimation of aerodynamic parameters from flight test. The consequent gains are increased flight-test safety, together with reduction of overall costs of development and certification.

Many applications of Artificial Neural Networks (ANN) in aircraft modeling (e.g.: Kim and Calise, 1997; Nϕrgaard et al, 1997; Ghosh, 1998; Raoi, 1994) have been developed. But the choice of the ANN architecture in these applications leads to a type of modeling where no physical meaning can be attached to the network parameters, hampering its ability to explicitly estimate the stability and control derivatives as they appear in the equations of motion. Although recurrent network architectures lead to a choice where this physical meaning can be attached, the application of this type architecture to practical cases has been considered very limited in scope, due to difficulties in tuning the network parameters, and due to its fixed structure (Raol and Jategaonkar, 1995).

One particular type of network architecture, known as Functional Link Network (FLN) (Chen and Billings, 1992) offers some advantages over other types. In the FLN, the hidden layer performs a functional expansion on the inputs, which gives the possibility to attach a physical meaning to the network parameters. This property brings the ability of direct estimation of the stability and control derivatives, as they appear in the equations of motion (Curvo, 2002). The approximation capability of an FLN depends on the chosen set of model bases that forms the hidden layer. Provided that the set of model bases is sufficiently rich (contains sufficient higher-order terms), any continuous function can be uniformly approximated to certain accuracy. Since the FLNs are linear in the parameters, these parameters can always be learned using an optimal linear estimation method, with noise incorporated in the observation data. Also, their structure facilitates the use of pruning techniques during training, to control to a minimum the number of neurons and connections.

Here, the representation capabilities of the FLN are explored in the problem of aircraft aerodynamic modeling. The resulting method can be seen as an extension of the usual Equation Error Method (see e.g., Hamel and Jategaonkar, 1996) as far as three aspects are concerned: first; the possibilities offered by the use of a FLN as consequence of the freedom in the choice of base function set; second, the use of a stochastic optimal linear estimation algorithm, more specifically the Kalman filtering that naturally allows the treatment of measurement noise in the dependent variables; and third, the optimization of the neural model, in terms of size and complexity, through the use of a pruning technique.

This paper follows a previous one (Curvo and Rios Neto, 2001) and presents a complete and detailed version of the proposed method. The method has been tested in the example problem of identifying the longitudinal dynamic model of a high performance aircraft (Stevens and Lewis, 1992). It is organized in 5 sections. Following this Introduction, Section 2 gives a comprehensive treatment; where the modeling, training, and structure sizing of the FLN are explained. Section 3 presents the example problem, chosen for demonstration of the effectiveness of the method. Section 4 presents a discussion of test results. Finally, Section 5 presents the conclusions.

2. Modeling with Functional Link Networks

The FLNs are composed of elements arranged in three layers: an input layer, of fan out elements; one hidden layer, of base functions; and an output layer of linear combination of the base functions. The general architecture of an FLN is shown in Figure (1). The hidden layer, is the one responsible for transforming the input n dimensional set into a p dimensional functional expansion set (Narendra and Parthasarathy, 1990; Chen and Billings 1992), that is:

$$\mathbf{x} \in R^n \rightarrow [h_1(\mathbf{x}) \dots h_p(\mathbf{x})]^T \in \Re^p \tag{1}$$

The output layer is composed of m nodes, each one being a linear combiner, leading to the input-output parameterized mapping:

$$\hat{\mathbf{f}} : \Re^n \rightarrow \Re^m \tag{2}$$

$$\hat{f}_j(\mathbf{x}, \mathbf{w}) = \sum_{k=1}^{p} w_{jk} h_k(\mathbf{x}) \qquad 1 \le j \le m$$

$$\hat{\mathbf{y}}(t) = \hat{\mathbf{f}}[\mathbf{x}(t), \mathbf{w}] \tag{3}$$

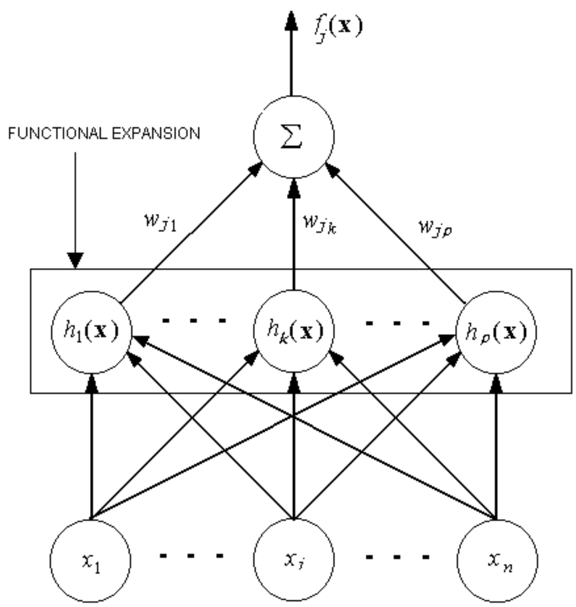


Figure 1 –Functional link network.

If an FLN should approximate a continuous mapping,

$$\mathbf{f} \in \mathbf{C} : \mathbf{x} \in \mathbf{D} \subset \Re^n \rightarrow \mathbf{y} \in \Re^m \tag{4}$$

it has to be trained. This training is done by adjustment of the vector of free network parameters, $\mathbf{w}$, such that the network is able to fit a given input-output training set.

$$\Im = \{[\mathbf{x}(t), \mathbf{y}(t)] : \mathbf{y}(t) = \mathbf{f}[\mathbf{x}(t)],\ t = 1, 2, \dots, T\} \tag{5}$$

In the case of the FLN, this can be done separately for each component of the parameterized network mapping. Consider a generic component in Equation (3):

$$\hat{\mathbf{y}}_j(t) = \mathbf{H}(\mathbf{x}(t))\,\mathbf{w}_j \qquad\qquad 1 \le j \le m \tag{6}$$

The problem of training the network, based on a given training set, can be solved separately for each component j by minimizing the following functional form:

$$J_j(\mathbf{w}_j) \equiv \frac{1}{2}\left[(\mathbf{w}_j - \overline{\mathbf{w}}_j)^T \overline{\mathbf{P}}_j^{-1}(\mathbf{w}_j - \overline{\mathbf{w}}_j)\right] + \frac{1}{2}\sum_{t=1}^{T}\left[(\mathbf{y}_j(t) - \mathbf{H}(x(t))\mathbf{w}_j)^T \mathbf{R}_j^{-1}(t)(\mathbf{y}_j(t) - \mathbf{H}(x(t))\mathbf{w}_j)\right] \tag{7}$$

where the inverses of $\overline{\mathbf{P}}_j$ and $\mathbf{R}_j$ are weight matrices, related to the quality of the initial estimates of the network parameters and training data set respectively. If a stochastic meaning is attached to these matrices, this minimization is equivalent to solve the following Stochastic Linear Parameter Estimation Problem (Rios Neto, 1997),

$$\begin{aligned} \overline{\mathbf{w}}_j &= \mathbf{w}_j + \overline{\mathbf{e}}_j \\ \mathbf{y}_j(t) &= \mathbf{H}(\mathbf{x}(t))\mathbf{w}_j + \mathbf{v}_j(t) \end{aligned} \qquad t = 1, 2, \ldots, T \ , \ j = 1, 2, \ldots, m \tag{8}$$

$$\begin{aligned} E[\mathbf{v}_j(t)] &= 0 \qquad & E[\mathbf{v}_j^2(t)] &= \mathbf{R}_j(t) \\ E[\overline{\mathbf{e}}_j] &= 0 \qquad & E[\overline{\mathbf{e}}_j \cdot \overline{\mathbf{e}}_j^T] &= \overline{\mathbf{P}}_j \end{aligned} \tag{9}$$

which can be solved by the following recursive linear estimation algorithm (Gelb, 1996):

$$\begin{aligned} &\mathbf{K}_j(t) = \overline{\mathbf{P}}_j \cdot \mathbf{H}^T(x(t))\left[\mathbf{R}_j(t) + \mathbf{H}\ (\mathbf{x}(t))\overline{\mathbf{P}}_j\mathbf{H}^T(\mathbf{x}(t))\right]^{-1} \\ &\mathbf{P}_j(t) = \left[\mathbf{I}_j - \mathbf{K}_j(t)\mathbf{H}\ (x(t))\right]\overline{\mathbf{P}}_j(t), \quad \overline{\mathbf{P}}_j(t) = \mathbf{P}_j(t-1), \quad \overline{\mathbf{P}}_j(0) = \overline{\mathbf{P}}_j \\ &\hat{\mathbf{w}}_j(t) = \overline{\mathbf{w}}_j(t) + \mathbf{K}_j(t)(\mathbf{y}_j(t) - \mathbf{H}(x(t))\overline{\mathbf{w}}_j(t)), \quad \overline{\mathbf{w}}_j(t) = \hat{\mathbf{w}}_j(t-1), \quad \overline{\mathbf{w}}_j(0) = \overline{\mathbf{w}}_j \end{aligned} \tag{10}$$

Choosing a proper network size is of great practical importance, smaller networks mean consuming less time in network predictions. It is thus important the use of pruning techniques for the elimination of the least significant weights, which do not affect the accuracy, and the ability of generalization of the network. The Optimal Brain Surgeon (OBS) algorithm, proposed by Hassibi and Wolff (1993), provides a good framework for the elimination of redundant network parameters. The algorithm relies on a local approximation of the learning error $\hat{\sigma}$ in the vicinity of $\hat{\mathbf{w}}$.

$$\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{y}(t) - \mathbf{H}(x(t))\hat{\mathbf{w}}_j)^T(\mathbf{y}(t) - \mathbf{H}(x(t))\hat{\mathbf{w}}) \tag{11}$$

$$\delta(\hat{\sigma}^2) = \frac{\partial(\hat{\sigma}^2)}{\partial\hat{\mathbf{w}}}\delta\hat{\mathbf{w}} + \frac{1}{2}\delta\hat{\mathbf{w}}^T\frac{\partial^2(\hat{\sigma}^2)}{\partial\hat{\mathbf{w}}\partial\hat{\mathbf{w}}}\delta\hat{\mathbf{w}} + O^s \tag{12}$$

where $\hat{\mathbf{w}}^T = [\hat{\mathbf{w}}_1^T, \hat{\mathbf{w}}_2^T \ldots, \hat{\mathbf{w}}_m^T]$.

Since this approximation is made in the vicinity of a minimum, in the space of network parameters, the fist term in Equation (12) vanishes. The higher order terms are not taken into account, so that:

$$\delta(\hat{\sigma}^2) = \frac{1}{2}\delta\hat{\mathbf{w}}^T\frac{\partial^2(\hat{\sigma}^2)}{\partial\hat{\mathbf{w}}\partial\hat{\mathbf{w}}}\delta\hat{\mathbf{w}} \tag{13}$$

The elimination of a given component w_{jk} in the vector of parameters is given by:

$$\mathbf{e}_{jk}^{T}\,\delta\hat{\mathbf{w}} + \hat{w}_{jk} = 0 \tag{14}$$

where $\mathbf{e}_{jk}$ is a unit vector corresponding, in terms of parameter space, to a component w_{jk}. The OBS algorithm solves the following optimization problem:

$$\min_{jk}\left\{ \min_{\delta\hat{\mathbf{w}}}\left(\frac{1}{2}\delta\hat{\mathbf{w}}^{T}\mathbf{A}\,\delta\hat{\mathbf{w}} \right) \middle| \mathbf{e}_{jk}^{T}\delta\hat{\mathbf{w}} + \hat{w}_{jk} = 0 \right\} \tag{15}$$

with $\mathbf{A} = \partial^{2}\left(\hat{\sigma}^{2}\right)/\partial\hat{\mathbf{w}}\partial\hat{\mathbf{w}}$.

The solution of this problem is given by

$$\delta\hat{\mathbf{w}} = -\frac{\hat{w}_{jk}^{2}}{\left(\mathbf{e}_{jk}^{T}\mathbf{A}^{-1}\,\mathbf{e}_{jk}\right)}\mathbf{A}^{-1}\,\mathbf{e}_{jk} \tag{16}$$

$$S_{jk} = \frac{1}{2}\frac{\hat{w}_{jk}}{\left(\mathbf{e}_{jk}^{T}\mathbf{A}^{-1}\,\mathbf{e}_{jk}\right)} \tag{17}$$

Equation (16) provides the correction to be made in the network weight parameters after the removal of a component w_{jk}, and Equation (17) is an indicator of the increase in the network approximation error, caused by the removal of that component. In practice, the pruning procedure is done as follows:

1. Start training the network totally connected, using a sufficiently rich set of base functions;
2. Apply Equation (17) to find the weight producing the least error after being eliminated; using Equation (16) do the correction in the vector of weights;
3. Use a mean prediction error (MPE) criterion, in a test data set; if it is not possible to decrease the MPE anymore, retrain the network with the remaining connections and stop; otherwise recalculate **A,** and go back to (2).

3. Demonstration Example

The example problem adopted for demonstration of the proposed technique consists of a nonlinear longitudinal model of a high performance aircraft. The aerodynamic model used is a classical model in table format that incorporates some of the nonlinear characteristics for this type of airplane. A more complete description of this model is found at Stevens and Lewis (1992).

Body Normal Force:

$$Z = \bar{q}S_{w}\,CZ \tag{18}$$

$$CZ = CZ_{0}(\alpha,\beta,\tilde{q},\delta_{e}) + CZ_{\alpha}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\alpha + CZ_{\delta_{e}}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\delta_{e} + CZ_{q}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\tilde{q} + \cdots + O^{S} \tag{19}$$

Body Longitudinal Force:

$$X = \bar{q}S_{w}\,CX \tag{20}$$

$$CX = CX_{0}(\alpha,\beta,\tilde{q},\delta_{e}) + CX_{\alpha}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\alpha + CX_{\delta_{e}}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\delta_{e} + CX_{q}(\alpha,\beta,\tilde{q},\delta_{e})\Delta\tilde{q} + \cdots + O^{S} \tag{21}$$

Body Pitching Moment:

$$M = \bar{q}S_{w}\bar{c}_{w}\,Cm \tag{22}$$

$$Cm = Cm_0(\alpha,\beta,\tilde{q},\delta_e) + Cm_\alpha(\alpha,\beta,\tilde{q},\delta_e)\Delta\alpha + Cm_{\delta_e}(\alpha,\beta,\tilde{q},\delta_e)\Delta\delta_e + Cm_q(\alpha,\beta,\tilde{q},\delta_e)\Delta\tilde{q} + \cdots + O^S \tag{23}$$

where:

$\tilde{q} = q\bar{c}/2V_R$.

$\bar{q}$	- dynamic pressure;
S_w	- wing area;
$\bar{c}_w$	- mean aerodynamic chord of the wing;
α	- angle of attack (see Fig 2);
δ_e	- elevator deflection (see Fig 2);
q	- pitching velocity (see Fig 2);
V	- airspeed.

Compressibility effects are not taken into account. The corresponding dynamic model, relative to an inertial frame, is a nonlinear longitudinal power-on model, described by the following set of equations (Etkin, 1972; Nelson, 1990; Stevens and Lewis, 1992; Durham, 1998).

Force Equation

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} r(v) - q(w) \\ -r(u) + p(w) \\ q(u) - p(v) \end{bmatrix} + \frac{1}{m}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \frac{1}{m}\begin{bmatrix} F_{gX} \\ F_{gY} \\ F_{gZ} \end{bmatrix} + \frac{1}{m}\begin{bmatrix} F_{TX} \\ F_{TY} \\ F_{TZ} \end{bmatrix} \tag{24}$$

Moment Equation

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (c_1\, r + c_2\, p)q + c_3\, \bar{L} + c_4\, N \\ c_5\,(p\, r) - c_6\,(p^2 - r^2) + c_7\, M \\ (c_8\, p - c_2\, r)q + c_4\, \bar{L} + c_9\, N \end{bmatrix} \tag{25}$$

where:

X, Y, and Z	– aerodynamic forces (longitudinal, lateral, and normal);
Fg_X, Fg_Y, and Fg_Z	– gravity force components;
F_{TX}, F_{TY}, and F_{TZ}	– thrust force components;
p, q, and r	– angular velocity components (roll velocity, pitch velocity, and yaw velocity);
u, v, and w	– linear velocity components (longitudinal, normal, and lateral);
m	– aircraft mass;

and

$$c_1 = \frac{(I_{YY} - I_{ZZ})I_{ZZ} - I_{XZ}^2}{(I_{XX} \cdot I_{ZZ} - I_{XZ}^2)} \quad c_2 = \frac{(I_{XX} - I_{YY} + I_{ZZ})I_{XZ}}{(I_{XX} \cdot I_{ZZ} - I_{XZ}^2)} \quad c_3 = \frac{I_{ZZ}}{(I_{XX} \cdot I_{ZZ} - I_{XZ}^2)}$$

$$c_4 = \frac{I_{XZ}}{(I_{XX} \cdot I_{ZZ} - I_{XY}^2)} \quad c_5 = \frac{(I_{ZZ} - I_{XX})}{I_{YY}} \quad c_6 = \frac{I_{XZ}}{I_{YY}}$$

$$c_7 = \frac{1}{I_{YY}} \quad c_8 = \frac{(I_{XX} - I_{YY})I_X + I_{XZ}^2}{(I_{XX} \cdot I_{ZZ} - I_{XZ}^2)} \quad c_9 = \frac{I_{XX}}{(I_{XX} \cdot I_{ZZ} - I_{XZ}^2)}$$

The inertia tensor relative to the body system is given by

$$\mathbf{I} = \begin{bmatrix} I_{XX} & 0 & -I_{XZ} \\ 0 & I_{YY} & 0 \\ -I_{ZX} & 0 & I_{ZZ} \end{bmatrix} \tag{26}$$

$I_{YZ} = I_{YX} = 0$ results from the hypothesis of body symmetry relative to the vertical plane (OX_BZ_B).

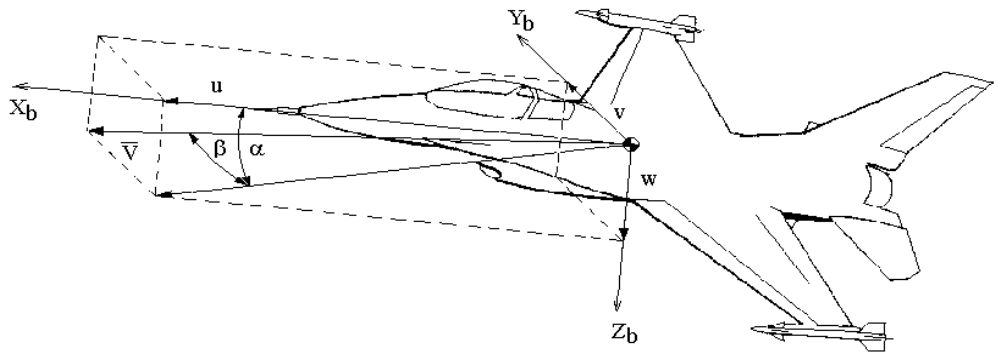


Figure 2 – Parameters of the longitudinal dynamics.

The state vector is thus defined by the following set of variables:

$$\mathbf{x}^T = [V \quad \beta \quad q \quad \delta_e] \tag{27}$$

The geometry, mass, and aerodynamic data of the airplane used in the calculations are the same as in Stevens and Lewis (1992), and Curvo (20002).

Wing Dimensions:

$$b_W = 9.144 \ m$$
$$S_W = 27.871 \ m^2$$
$$\bar{c} = 3.450 \ m$$

C.G. reference position:

$$X_{cg} = 0.35 \ \% \bar{c}$$

Angular moment due to engine:

$h_E = 217.158 \ Kg \ m^2 / s$ (assumed to be constant)

Thrust line position relative to C.G.

$$X_{Thr} = 0.000 \ m$$
$$Y_{Thr} = 0.000 \ m$$
$$Z_{Thr} = 0.000 \ m$$

Mass Properties:

$$m = 92.180\ Kg$$

$$I_{XX} = 12888.301\ Kg\ m^2$$

$$I_{YY} = 75752.700\ Kg\ m^2$$

$$I_{ZZ} = 85641.513\ Kg\ m^2$$

$$I_{XZ} = 1332.805\ Kg\ m^2$$

Simulated data, instead of flight test data, is used to estimate the coefficients of the aerodynamic model equations, namely: $CX_0, CX_\alpha, CX_q, CX_{\delta_e}, CZ_0, CZ_\alpha, CZ_q, CZ_{\delta_e}, Cm_0, Cm_\alpha, Cm_q, Cm_{\delta_e}$, and higher order terms. The use of simulated data does not invalidate the results, since the objective is to test a procedure and demonstrate the use of an FLN in estimating these coefficients.

The FLN adopted emulates the aerodynamic model structure of Equations (19), (21) and (23). The inputs to the first layer of fan out elements (see Figure 1) are formed by the following set of variables: airspeed (V), pitch velocity (q), angle of attack (α), angle of sideslip (β), and elevator deflection (δ_e) - this last parameter being a deterministic control variable.

The aerodynamic neural model construction starts with a minimum size RNA, made up of a set of parameters representative of a first order aerodynamic model. These parameters are nothing more than the stability and control derivatives; whose approximate values were evaluated using the aerodynamic data bank as guideline. Other alternative would be the use of estimated theoretic values. For the a priori errors in these coefficients (to approximate as a diagonal the matrix of a priori covariance in estimation errors, in Equation (9),) the following criterion, as suggested by Bauer and Andrisani (1990), was considered: $\bar{\boldsymbol{\sigma}}_{jk}^2 \approx \left(\bar{w}_{jk}/4\right)^2$.

Once established, this initial model is expanded one term at a time, and each time a new term is added the network is retrained. The initial value for the covariance of the additional higher order terms is assumed to be equal to the estimated covariance of the previous estimated model. This approximation was adopted in face of a lack of adequate criteria for the definition of more precise values for the covariance of these higher order terms. The procedure is repeated until a network of sufficient size, totally connected and with acceptable error norm is obtained. Starting with the fully connected network, the optimal brain surgeon algorithm (OBS) is used in the network optimization process, where the least significant parameters are eliminated, up to a point where an optimum balance between a rigid and flexible model is attained. Here, flexible model means that it lacks generalization capacity due to under modeling, (unable to capture the non-linearity) and rigid model means that it lacks generalization capacity due to over modeling (modeling noise output noise) (Curvo, 2002).

The aerodynamic coefficients, used as dependent variables in the network training set $\Im = \{[\mathbf{x}(t), \mathbf{y}(t)] : \mathbf{y}(t) = \mathbf{f}[\mathbf{x}(t)],\ t = 1, 2, \ldots, T\}$, are calculated from the dependent variables (sensors outputs), engine thrust, aircraft mass properties, and atmospheric properties, Equations 28, 29, and 30 (Klein, 1981).

$$CX = \frac{mg}{\bar{q}S_W}\left(a_x - \frac{F_{TX}}{mg}\right) \tag{28}$$

$$CZ = \frac{mg}{\bar{q}S_W}a_z \tag{29}$$

$$Cm = \frac{I_{YY}}{\bar{q}S_W\bar{c}}\left[\dot{q} - \left(\frac{I_{ZZ} - I_{XX}}{I_{YY}}\right)pr - \frac{I_{XZ}}{I_{YY}}\left(r^2 - p^2\right)\right] \tag{30}$$

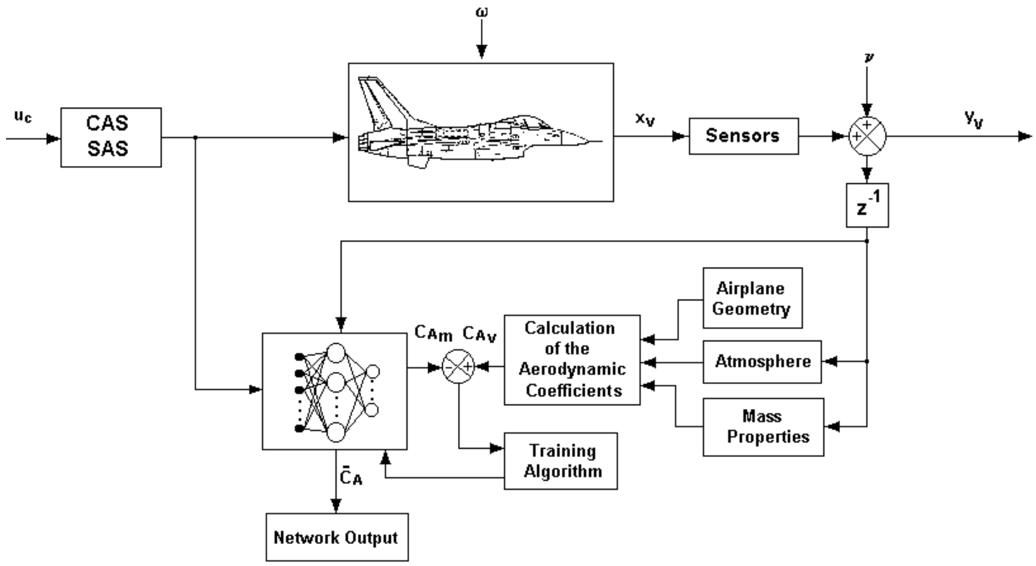


Figure 3 – Network training scheme.

The initial values of the network parameters, representing the first order terms (stability and control derivatives) for the longitudinal aerodynamic model, used as the initial values for the network model assembly, and its respective variances are listed in Table 1. The measurement errors are modeled as white noise with normal distribution of zero average. The measurement variance assumed for the sensors are listed in Table 2 (Bauer and Andrisani, 1990; Napolitano et al, 2000). Plant disturbances are included in the simulations, using the Von Karman model for atmospheric turbulence. These disturbances are taken into account through the effect of atmospheric turbulence on the airspeed components; the effects on angular velocities are not taken into account (Nelson, 1990).

TABLE 1 – INITIAL PARMETER VALUES FOR THE LONGITUDINAL AERODYNAMIC MODEL

	CX	σ_{CX}	*CZ*	σ_{CZ}	*Cm*	σ_{Cm}
w(*j*,0)	-0.02100	±5.250E-03	-0.10000	±2.500E-02	-0.00900	±2.250E-03
w(*j*,1)	0.09168	±2.292E-02	-3.76461	±9.400E-01	0.08595	±2.149E-02
w(*j*,2)	0.30800	±7.700E-02	-28.90000	±7.225E+00	-5.23000	±1.308E+00
w(*j*,3)	0.00287	±7.175E-04	-0.43548	±1.100D-01	-0.65322	±1.633E-01

TABLE 2 – SENSOR MESUREMENT ERROR DATA

Parameters	Variances
Angular Velocities (*p*, *q* e *r*)	0.14°/s
Euler Angles (θ, ϕ e ψ)	0.57°
Acceleration components (a_x, a_y e a_z)	0.01 m/s^2
Airspeed (V_R)	3.35 m/s
Altitude (*h*)	3.0 m
Angles of Attack and Sideslip (α e β)	0.1°

Adapted from Napolitano et al (2000, p.114)

4. Results and Discussions

The complete set of base functions in the hidden layer of the fully connected network is listed in Table 3. The estimated network parameter data is summarized in Tables 4 through 6, where the parameters of fully connected, and optimized networks are shown.

TABLE 3 – BASE FUNCTION SET FOR LONGITUDINAL MODEL

$h_k(x)$					
$h_0=1.0$	$h_1=\alpha$	$h_2=\tilde{q}$	$h_3=\delta_e$		
	$h_4=\alpha^2$	$h_5=\alpha\,\tilde{q}$	$h_6=\alpha\,\delta_e$		$h_7=\alpha\,\beta$
	$h_8=\alpha^3$	$h_9=\alpha^2\,\tilde{q}$	$h_{10}=\alpha^2\,\delta_e$	$h_{11}=\alpha\,\delta_e^2$	
	$h_{12}=\alpha^4$	$h_{13}=\alpha^3\,\tilde{q}$	$h_{14}=\alpha^3\,\delta_e$	$h_{15}=\alpha\,\delta_e^3$	
	$h_{16}=\alpha^5$	$h_{17}=\alpha^4\,\tilde{q}$	$h_{18}=\alpha^4\,\delta_e$	$h_{19}=\alpha\,\delta_e^4$	
	$h_{20}=\alpha^6$	$h_{21}=\alpha^5\,\tilde{q}$	$h_{22}=\alpha^5\,\delta_e$	$h_{23}=\alpha\,\delta_e^5$	
	$h_{24}=\alpha^7$	$h_{25}=\alpha^6\,\tilde{q}$	$h_{26}=\alpha^6\,\delta_e$	$h_{27}=\alpha\,\delta_e^6$	
	$h_{28}=\alpha^8$				
	$h_{29}=\alpha^9$				

TABLE 4 – PARAMETERS FOR THE LONGITUDINAL FORCE COEFFICIENT MODEL (*CX*)

TOTALLY CONNECTED NETWORK			OPTMIZED NETWORK		
Total number of Parameters – 30 ASE = 0.98810E-03 (“Average Squared Error”) PSE = 0.11024E-02 (“Prediction Square Error”)			Total number of Parameters – 11 ASE = 0.99972E-03 (“Average Squared Error”) PSE = 0.10417E-02 (“Prediction Square Error”)		
Parameters		σ_w	Parameters		σ_w
$w_{1\,0}$	-0.21652E-01	±0.91754E-04	$w_{1\,0}$	-0.23399E-01	±0.91586E-04
$w_{1\,1}$	0.21579E+00	±0.11186E-02	$w_{1\,1}$	0.21809E+00	±0.10963E-02
$w_{1\,2}$	0.21491E+01	±0.53173E-01	$w_{1\,2}$	0.29463E+01	±0.53157E-01
$w_{1\,3}$	0.59486E-02	±0.69358E-03	$w_{1\,4}$	0.75492E+00	±0.69685E-03
$w_{1\,4}$	0.75708E+00	±0.47942E-02	$w_{1\,7}$	-0.26391E+00	±0.58114E-02
$w_{1\,5}$	0.19724E+00	±0.37913E-01	$w_{1\,8}$	-0.67610E+00	±0.11037E-01
$w_{1\,6}$	0.23464E-01	±0.89911E-02	$w_{1\,10}$	-0.22741E+00	±0.96838E-02
$w_{1\,7}$	-0.25313E+00	±0.36339E-01	$w_{1\,11}$	-0.33768E+00	±0.20507E-01
$w_{1\,8}$	-0.55308E+00	±0.12197E-01	$w_{1\,12}$	-0.76418E+00	±0.14360E-01
$w_{1\,9}$	0.66139E-01	±0.38268E-01	$w_{1\,24}$	0.14147E+01	±0.28723E-01
$w_{1\,10}$	-0.22361E+00	±0.21330E-01	$w_{1\,28}$	-0.78140E+00	±0.24627E-01
$w_{1\,11}$	-0.59363E+00	±0.24075E-01			
$w_{1\,12}$	-0.86008E+00	±0.20797E-01			
$w_{1\,13}$	0.24789E-01	±0.38347E-01			
$w_{1\,14}$	-0.21473E+00	±0.29123E-01			
$w_{1\,15}$	0.48043E-01	±0.33493E-01			
$w_{1\,16}$	-0.21951E+00	±0.25899E-01			
$w_{1\,17}$	0.10342E-01	±0.38370E-01			
$w_{1\,18}$	-0.20062E+00	±0.32314E-01			
$w_{1\,19}$	0.41890E-02	±0.36895E-01			
$w_{1\,20}$	0.15878E+00	±0.28486E-01			
$w_{1\,21}$	0.48584E-02	±0.38378E-01			
$w_{1\,22}$	-0.17251E+00	±0.34365E-01			
$w_{1\,23}$	-0.20666E-01	±0.37995E-01			
$w_{1\,24}$	0.28065E+00	±0.30406E-01			
$w_{1\,25}$	0.25928E-02	±0.38381E-01			
$w_{1\,26}$	-0.13923E+00	±0.35757E-01			
$w_{1\,27}$	0.15018E-01	±0.38292E-01			
$w_{1\,28}$	0.27231E+00	±0.32210E-01			
$w_{1\,29}$	0.21698E+00	±0.33876E-01			

TABLE 5 – PARAMETERS FOR THE NORMAL FORCE COEFFICIENT MODEL (*CZ*)

TOTALLY CONNECTED NETWORK			OPTMIZED NETWORK		
Total number of Parameters – 30 ASE = 0.80405E-02 (“Average Squared Error”) PSE = 0.88076E-02 (“Prediction Square Error”)			Total number of Parameters – 19 ASE = 0.81077E-02 (“Average Squared Error”) PSE = 0.86457E-02 (“Prediction Square Error”)		
Parameters		σ_w	**Parameters**		σ_w
$w_{2\,0}$	-0.10026E+00	±0.19482E-03	$w_{2\,0}$	-0.96210E-01	±0.19408E-03
$w_{2\,1}$	-0.36748E+01	±0.29816E-02	$w_{2\,1}$	-0.35600E+01	±0.26264E-02
$w_{2\,2}$	-0.26868E+02	±0.15048E+00	$w_{2\,2}$	-0.28712E+02	±0.14902E+00
$w_{2\,3}$	-0.52193E+00	±0.67061E-02	$w_{2\,4}$	-0.47383E+00	±0.65368E-02
$w_{2\,4}$	0.31932E+00	±0.19895E-01	$w_{2\,5}$	0.59072E+00	±0.23619E-01
$w_{2\,5}$	0.89827E-01	±0.45533E+00	$w_{2\,7}$	-0.12104E+01	±0.39527E+00
$w_{2\,6}$	0.48085E+00	±0.48092E-01	$w_{2\,8}$	-0.79992E+00	±0.40325E-01
$w_{2\,7}$	-0.13285E+01	±0.24102E+00	$w_{2\,10}$	-0.42496E+01	±0.86854E-01
$w_{2\,8}$	-0.13453E+01	±0.84961E-01	$w_{2\,11}$	0.12480E+01	±0.83280E-01
$w_{2\,9}$	0.28906E+00	±0.58204E+00	$w_{2\,12}$	0.40124E+01	±0.23660E+00
$w_{2\,10}$	-0.20811E+01	±0.16816E+00	$w_{2\,14}$	0.17131E+01	±0.21165E+00
$w_{2\,11}$	-0.36260E+00	±0.13535E+00	$w_{2\,16}$	0.20471E+01	±0.18610E+00
$w_{2\,12}$	0.19737E+01	±0.21524E+00	$w_{2\,18}$	0.54045E+01	±0.26274E+00
$w_{2\,13}$	0.12155E+00	±0.60666E+00	$w_{2\,20}$	-0.60755E+01	±0.58583E+00
$w_{2\,14}$	0.19724E+00	±0.34827E+00	$w_{2\,22}$	0.24622E+01	±0.35135E+00
$w_{2\,15}$	0.14313E+01	±0.37302E+00	$w_{2\,24}$	0.33374E+01	±0.42929E+00
$w_{2\,16}$	0.29383E+00	±0.35153E+00	$w_{2\,26}$	-0.60422E+01	±0.45301E+00
$w_{2\,17}$	-0.62307E-01	±0.61226E+00	$w_{2\,28}$	-0.14641E+01	±0.46978E+00
$w_{2\,18}$	0.14359E+01	±0.44821E+00	$w_{2\,29}$	1.92758E+00	±0.46135E+00
$w_{2\,19}$	0.17454E+01	±0.54846E+00			
$w_{2\,20}$	0.56175E+00	±0.43314E+00			
$w_{2\,21}$	-0.16356E+00	±0.61384E+00			
$w_{2\,22}$	0.25047E+01	±0.49250E+00			
$w_{2\,23}$	-0.20099E+01	±0.59529E+00			
$w_{2\,24}$	0.15056E+01	±0.46064E+00			
$w_{2\,25}$	-0.19654E+00	±0.61440E+00			
$w_{2\,26}$	0.32891E+01	±0.51983E+00			
$w_{2\,27}$	0.12742E+01	±0.60962E+00			
$w_{2\,28}$	0.20960E+01	±0.47195E+00			
$w_{2\,29}$	0.22064E+01	±0.48537E+00			

TABLE 6 – PARAMETERS FOR THE LONGITUDINAL MOMENT COEFFICIENT MODEL (*CM*)

TOTALLY CONNECTED NETWORK			**OPTMIZED NETWORK**		
Total number of Parameters – 30 ASE = 0.63427E-04 (“Average Squared Error”) PSE = 0.70272E-04 (“Prediction Square Error”)			Total number of Parameters – 23 ASE = 0.63434E-04 (“Average Squared Error”) PSE = 0.68117E-04 (“Prediction Square Error”)		
Parameters		σ_w	**Parameters**		σ_w
$w_{3\,0}$	-0.81953E-02	±0.17150E-05	$w_{3\,0}$	-0.85767E-02	±0.17152E-05
$w_{3\,1}$	0.88808E-01	±0.25463E-04	$w_{3\,1}$	0.97282E-01	±0.25568E-04
$w_{3\,2}$	-0.49644E+01	±0.12448E-02	$w_{3\,2}$	-0.47275E+01	±0.12430E-02
$w_{3\,3}$	-0.46817E+00	±0.56753E-04	$w_{3\,3}$	-0.52573E+00	±0.57363E-04
$w_{3\,4}$	-0.46233E+00	±0.15740E-03	$w_{3\,4}$	-0.45393E+00	±0.17012E-03
$w_{3\,5}$	-0.21153E+01	±0.34527E-02	$w_{3\,5}$	-0.12003E+01	±0.34429E-02
$w_{3\,6}$	0.55430E+00	±0.35904E-03	$w_{3\,6}$	-0.95045E-02	±0.40334E-03
$w_{3\,7}$	-0.13982E+00	±0.15380E-02	$w_{3\,8}$	0.11067E+01	±0.71911E-03
$w_{3\,8}$	0.77364E+00	±0.61574E-03	$w_{3\,9}$	-0.94598E+00	±0.82355E-03
$w_{3\,9}$	-0.10631E+01	±0.48737E-02	$w_{3\,10}$	-0.33857E-01	±0.19665E-02
$w_{3\,10}$	0.15755E+01	±0.12022E-02	$w_{3\,11}$	-0.36472E+00	±0.11285E-02
$w_{3\,11}$	-0.63037E+00	±0.10453E-02	$w_{3\,12}$	-0.12656E+01	±0.12842E-02
$w_{3\,12}$	-0.92808E+00	±0.14996E-02	$w_{3\,13}$	0.34572E+00	±0.19648E-02
$w_{3\,13}$	-0.48640E+00	±0.53332E-02	$w_{3\,14}$	-0.12117E+00	±0.37606E-02
$w_{3\,14}$	-0.18055E+01	±0.24817E-02	$w_{3\,15}$	-0.32410E+01	±0.25121E-02
$w_{3\,15}$	-0.37400E+01	±0.25793E-02	$w_{3\,16}$	0.63555E+00	±0.18459E-02
$w_{3\,16}$	0.51669E+00	±0.25304E-02	$w_{3\,18}$	0.15847E+01	±0.29193E-02
$w_{3\,17}$	-0.21186E+00	±0.54723E-02	$w_{3\,19}$	0.39908E+01	±0.45210E-02
$w_{3\,18}$	-0.87951E+00	±0.33216E-02	$w_{3\,22}$	0.17926E+01	±0.34996E-02
$w_{3\,19}$	0.69302E+00	±0.41296E-02	$w_{3\,23}$	0.64052E+00	±0.45966E-02
$w_{3\,20}$	0.35006E+00	±0.33123E-02	$w_{3\,26}$	-0.25719E+01	±0.35143E-02
$w_{3\,21}$	-0.78397E-01	±0.55223E-02	$w_{3\,27}$	-0.42104E+00	±0.54666E-02
$w_{3\,22}$	0.14157E+01	±0.36958E-02	$w_{3\,29}$	0.81052E-01	±0.28441E-02
$w_{3\,23}$	0.57608E+00	±0.50137E-02			
$w_{3\,24}$	-0.22562E-01	±0.35839E-02			
$w_{3\,25}$	-0.14403E-01	±0.55438E-02			
$w_{3\,26}$	0.24926E+01	±0.39089E-02			
$w_{3\,27}$	-0.81961E+00	±0.53910E-02			
$w_{3\,28}$	0.11250E+00	±0.36418E-02			
$w_{3\,29}$	0.64375E+00	±0.37086E-02			

TABLE 7 – LONGITUDINAL NEURO MODEL STRUCTURE

FUNCTION	ESTRUCTURE
$CX(\alpha,\ \delta_e)$	$w_{1-0} + w_{1-1}\alpha + w_{1-4}\alpha^2 + w_{1-8}\alpha^3 + w_{1-10}\alpha^2\delta_e +$ $w_{1-11}\alpha\,\delta_e{}^2 + w_{1-12}\alpha^4 + w_{1-24}\alpha^7 + w_{1-28}\alpha^8$
$\dfrac{\partial CX(\alpha)}{\partial\beta}$	$w_{1-7}\alpha$
$\dfrac{\partial CX(\alpha)}{\partial\tilde{q}}$	w_{1-2}
$CZ(\alpha,\ \delta_e)$	$w_{2-0} + w_{2-1}\alpha + w_{2-4}\alpha^2 + w_{2-8}\alpha^3 + w_{2-10}\alpha^2\,\delta_e + w_{2-11}\alpha\,\delta_e^2 +$ $w_{2-12}\alpha^4 + w_{2-14.}\alpha^3\,\delta_e + w_{2-16}\alpha^5 + w_{2-18}\alpha^4\,\delta_e + w_{2-20}\alpha^6 +$ $w_{2-22}\alpha^5\,\delta_e + w_{2-24}\alpha^7 + w_{2-26}\alpha^6\,\delta_e + w_{2-28}\alpha^8 + w_{2-29}\alpha^9$
$\dfrac{\partial CZ(\alpha)}{\partial\beta}$	$w_{2-7}\alpha$
$\dfrac{\partial CZ(\alpha)}{\partial\tilde{q}}$	$w_{2-2} + w_{2-5}\alpha$
$Cm(\alpha,\ \delta_e)$	$w_{3-0} + w_{3-1}\alpha + w_{3-3}\delta_e + w_{3-4}\alpha^2 + w_{3-6}\alpha\ \delta_e + w_{3-8}\alpha^3 +$ $w_{3-10}\alpha^2\,\delta_e + w_{3-11}\alpha\,\delta_e^2 + w_{3-12}\alpha^4 + w_{3-14}\alpha^3\,\delta_e + w_{3-15}\alpha\,\delta_e^3 +$ $w_{3-16}\alpha^5 + w_{3-18}\alpha^4\,\delta_e + w_{3-19}\alpha\,\delta_e^4 + w_{3-22}\alpha^5\,\delta_e + w_{3-23}\alpha\,\delta_e^5 +$ $w_{3-26}\alpha^6\delta_e + w_{3-27}\alpha\delta_e^6 + w_{3-29}\alpha^9$
$\dfrac{\partial Cm(\alpha)}{\partial\tilde{q}}$	$w_{3-2} + w_{3-5}\alpha + w_{3-9}\alpha^2 + w_{3-13}\alpha^3$

The simulated flight conditions at which the identification maneuvers were simulated correspond to an altitude of 500 meters (1650 ft) and reference airspeeds (trim points) spanning from 60 m/s (117 knots) to 240 m/s (470 knots). This speed variation is equivalent to an angle of attack variation from 22.50° to 0.15°. A total of ten trim points each one with a training set made up of 1000 training pairs were used. The time interval of each simulated maneuver was 30 seconds. Figure 4 illustrates the applied command sequence. Simulation results showing a comparison between the true model (data bank) end identified model (FLN) for the longitudinal response of the aircraft, representing a trim point for airspeed equals to 160 m/s are shown in Figures 5(a), 5(b), 6(a), and 6(b)

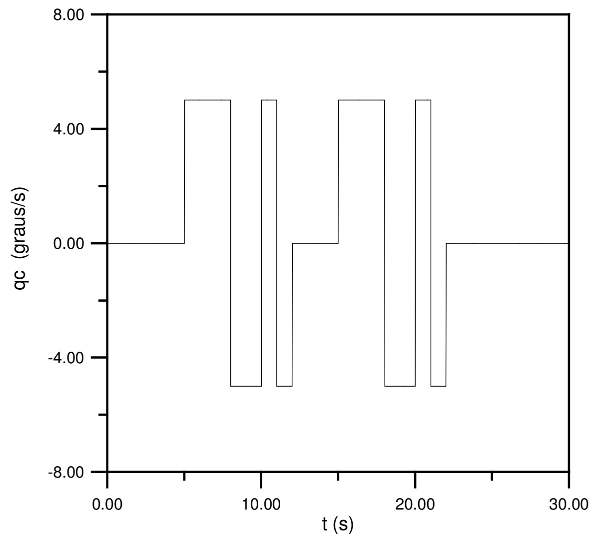


Figure 4 – Longitudinal command sequence for identification maneuvers

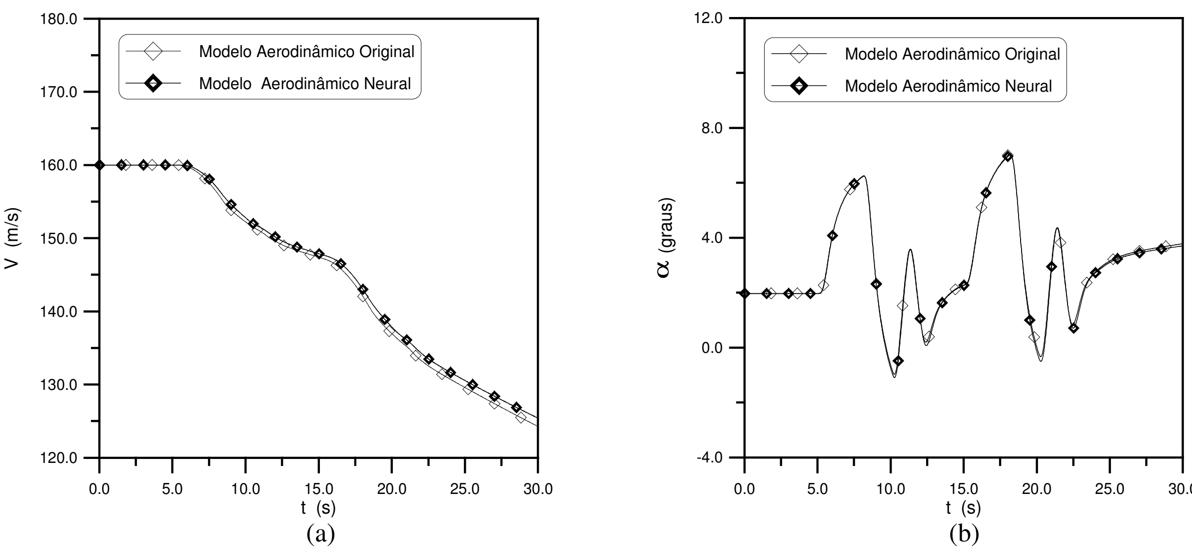
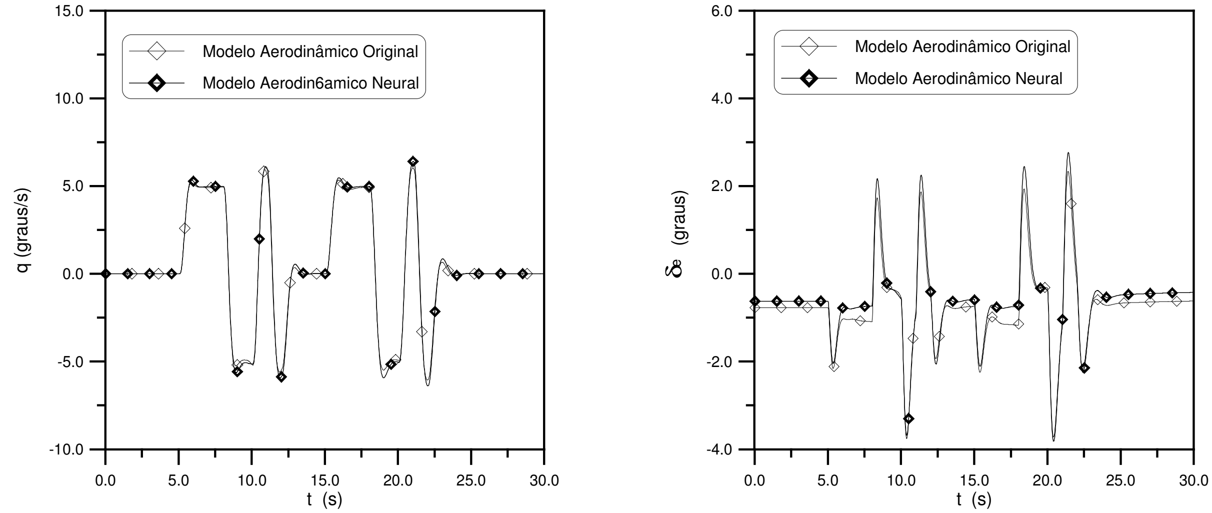


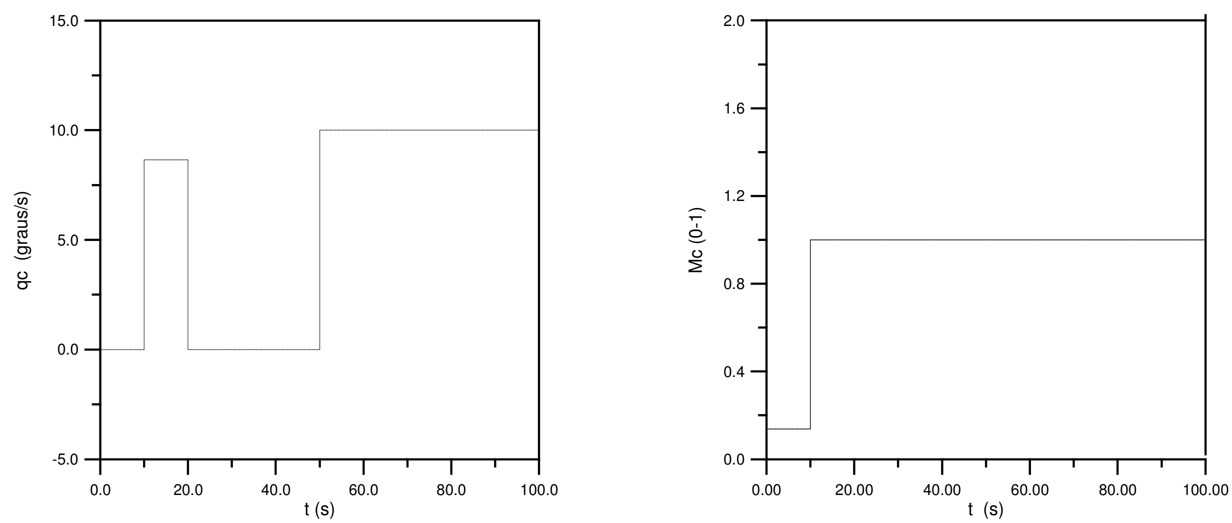
Figure 5 – Simulation 160 m/s: (a) airspeed and (b) angle of attack.



(a) (b)

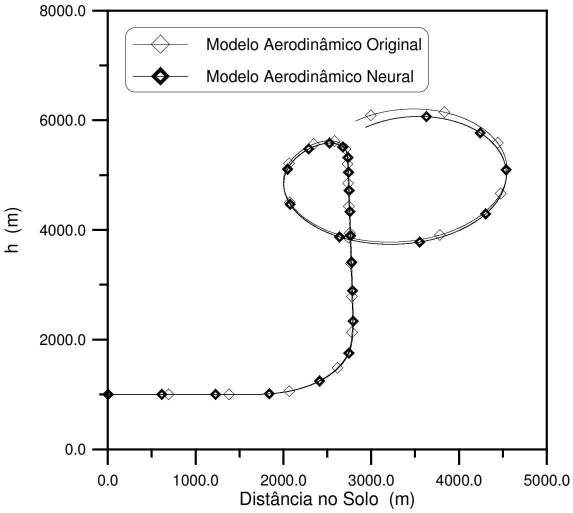
Figure 6 – Simulation 160 m/s: (a) pitch velocity and (b) elevator deflection.

These results are quite good and consistent but cannot be considered as final. It is necessary to test the generalization capacity of the identified model in simulation conditions different from de ones used for identification purpose. In order to test de model, a maneuver involving not only pitching command (elevator) but also thrust command, has been used. This maneuver, as opposed to the one used for identification, is characterized by a large variation of airspeed, altitude, and load factor. This maneuver is described at Stevens and Lewis (1992). It starts with the airplane trimmed at airspeed of 153 m/s (297 knots), at an altitude of 1000 meters (3300 ft), and cg position in 30% of mean aerodynamic chord. After 10 seconds of trimmed flight, a pitch command of 8.9 degree/s is applied during 5 seconds. At the same time maximum engine thrust is applied with the aircraft in a zoom climb attitude, Figure 7(a), and 7(b). After reaching an altitude of approximately 5400 meters (18000 ft), a looping maneuver is initiated with the application of a pitching command of 10 degree/s during the remaining 50 seconds of this maneuver, Figure 8(a). Time history for airspeed, pitch velocity, angle of attack, and elevator deflection are shown in Figures 8(b), 9(a), 9(b), and 10

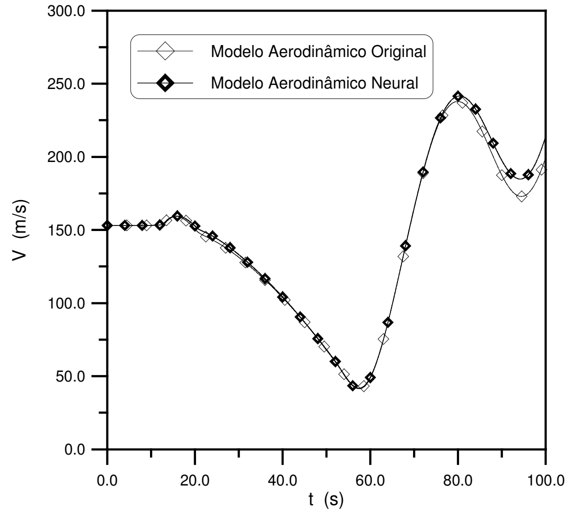


(a) (b)

Figure 7 – Test maneuver: (a) pitch command and (b) thrust command.

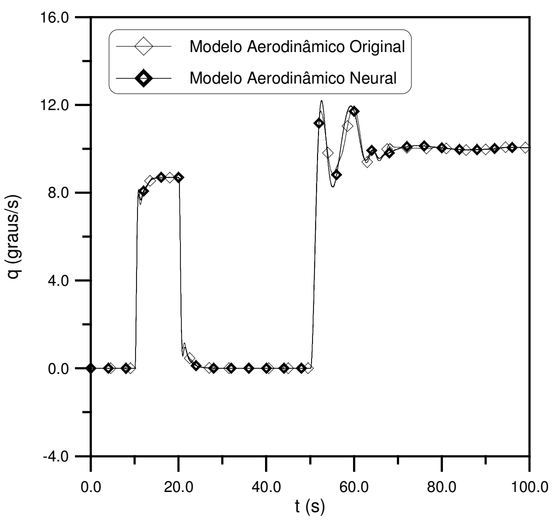


(a)

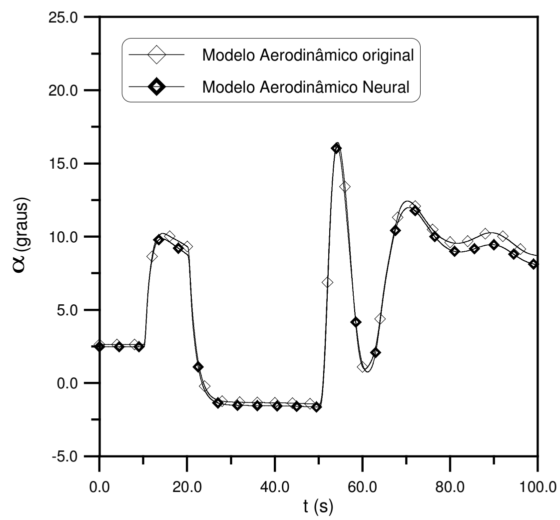


(b)

Figure 8 – Test maneuver: (a) vertical plane trajectory and (b) airspeed.



(a)



(b)

Figure 9 – Test maneuver: (a) pitch velocity and (b) angle of attack

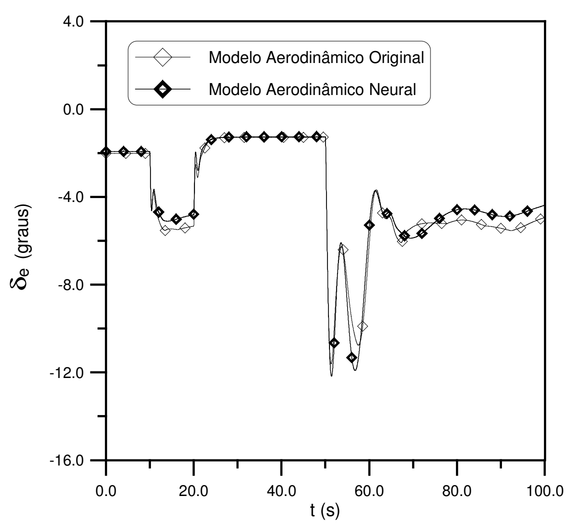


Figure 10 – Test maneuver: elevator deflection.

The results obtained are of good quality and indicate that the method is a competitive choice, when considered together with other estimation by filtering approach methods that also preserve the physical meaning of estimated parameters (Klein; 1981, Bauer and Andrisani, 1990; Curvo 2000). The pruning technique was able to find which connections could be disregarded, to get a simpler aerodynamic model. Though pre filtering of noisy measurements of independent variables was not used, the Kalman filtering estimation algorithm was still able to filter out the effect of this noise.

5. Conclusions

The adoption of Functional Link Networks combined with Optimal Linear Estimation and the Optimal Brain Surgeon pruning technique lead to a two-step method that can be seen as an extension of the usual Equation Error Method and an alternative to the Parameter Estimation by Filtering Approach Method. The proposed method has the advantageous characteristics of working with an aerodynamic model structure commonly used by aerodynamicists and also the ability to separate the aerodynamic parameter identification problem from state estimation. These characteristics are attained in a method capable of providing good parameter estimations, without the drawbacks of biased estimates of the Equation Error Method and tuning difficulties of the Filtering Method.

The Kalman recursive structure of the parameter estimation algorithm produced experimental results in the demonstration testing where the independent variables noise was effectively filtered. This may be due to the fact that oscillations in the Kalman gain do not affect results and that this noise was filtered together with the observation noise in the calculations with the observation residue (see Equation (10)). Anyway, even if this was not the case, the use of independent variables as inputs of the FLN should not be a limitation in face of present possibilities of separating the state estimation and of using high accuracy sensors.

The modeling with the FLN opens new possibilities in terms of better adjusting an aerodynamic model, by combining the enrichment of base functions with the elimination of least relevant connections provided by the pruning technique.

The combination of these characteristics makes the method a strong candidate for off line application in aircraft identification during certification campaigns. With the present onboard available processing capacity it is also a candidate for on line, in flight, aircraft identification.

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