

INTEGRATED GPS/INS NAVIGATION SYSTEM BASED ON A GYROSCOPE-FREE IMU

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Abstract: This paper analyses the performance of a low cost INS, based on a gyro-free IMU that only uses multiples accelerometers and is aided by a GPS. The IMU is composed by a specific array of accelerometers in such way that linear and angular accelerations can be computed. The GPS/INS loosely integration approach is implemented by a Kalman filter. The performance of the integrated system is assessed by using computer simulation.

Keywords: inertial navigation system, GPS integrated with INS, gyro-free inertial measurement unit.

1. INTRODUCTION

Inertial navigation systems are widely used in many applications including civilian and military aviation, spatial and nautical segments, automobiles, automated agricultural and construction vehicles, and robotics. Recent advances in MEMS technology have made inertial sensors more affordable and thus costs of micro-machined accelerometer and gyroscopes are decreasing while their performance characteristics are being improved [1], [2]. The INS gives the position, velocity and attitude but with growing time degradations due to sensor errors and random disturbances. Hence, an update or position fix can be taken from an external reference such as data from the GPS receptor. Micro-machined accelerometers are now in large-volume production, cost a few dollars and have been showing reliability, and this also happens to GPS receivers, making a gyro-free GPS/INS attractive for low-cost, medium performance application areas.

2. GYRO-FREE IMU CONFIGURATION

Standard inertial measurement unit, or IMU, uses accelerometers to sense linear accelerations and gyroscopes to sense angular velocity. A gyro-free IMU is a specific array of accelerometers where location and orientation are chosen in such way that angular and linear motions can be computed by using decoupled equations.

Since gyroscope's technology, besides it high cost, also suffers restrictions to be imported, efforts to develop a gyro-free IMU have been done. Gyro-free IMU have been described by Pandgaonkar et alli [3], Merhav [4], Trabasso [5] and Chen et alii [6]. Gyro-free inertial navigation

systems integrated with GPS are also described by Mostov et alii [7], Park e Tan [8] and Marques Filho [9].

In this paper a cube configuration with six accelerometers is used [6]. The array has one accelerometer at the center of each face of the cube and the sensing axes are along the respective cube diagonal as shown in Figure 1 (adapted from [6]).

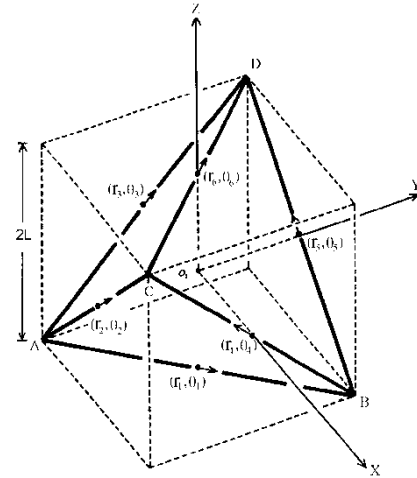


Fig. 1. Cube gyro-free IMU configuration

The equations of angular and linear accelerations are decoupled and are described with reference to the center of the cube, defined as the origin of the moving vehicle frame and are described by [6]:

$$\ddot{\mathbf{x}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{2\sqrt{2}} \cdot \mathbf{J}_1 \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}, \quad \dot{\mathbf{x}} = \int \ddot{\mathbf{x}} \quad (1)$$

$$\mathbf{f}^v = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \frac{1}{2\sqrt{2}} \cdot \mathbf{J}_2 \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} + \mathbf{I} \cdot \begin{bmatrix} \omega_y \cdot \omega_z \\ \omega_z \cdot \omega_x \\ \omega_x \cdot \omega_y \end{bmatrix} \quad (2)$$

where:

A_1, \dots, A_6 are the accelerometers outputs.

$\dot{\omega}_{iv}^v$ are the angular accelerations of a vehicle frame (v) with relation to an inertial frame (i), and expressed in the vehicle frame.

\mathbf{f}^v are the linear accelerations or specific force of the vehicle frame (v).

\mathbf{J}_1 and \mathbf{J}_2 are matrices that define the accelerometer's location and orientation on cube faces and are described by:

$$\mathbf{J}_1 = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 & 1 & 0 \end{bmatrix}, \quad (3)$$

$$\mathbf{J}_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \quad (4)$$

3. GPS/INS INTEGRATION FILTER DESIGN

3.1 Inertial navigation equations

Strapdown inertial navigation algorithms consist of attitude, velocity and position equations that can be described in several reference frames. The equations for the local tangent plane (LTP), or navigation frame, can be written as [10]:

$$\dot{\mathbf{r}}^n = \mathbf{v}^n, \quad \dot{\mathbf{v}}^n = \mathbf{f}^n - \mathbf{g}^n, \quad \dot{\mathbf{e}}^n = \mathbf{\Omega}^n \mathbf{e}^n \quad (5)$$

where:

$\mathbf{r}^n = \{p, \lambda, h\}$ is the position vector for latitude, longitude and height components,

$\mathbf{v}^n = \{\dot{p}, \dot{\lambda}, \dot{h}\}$ is the velocity vector,

\mathbf{f}^n is the specific force vector, where $\mathbf{f}^n = \mathbf{C}_v^n \cdot \mathbf{f}^v$

\mathbf{C}_v^n is a transformation matrix from vehicle to navigation frame,

\mathbf{g}^n is the gravity vector, and

$\mathbf{\Omega}^n$ denotes an angular velocity matrix.

3.2 Inertial navigation error model

The Kalman Filtering technique used to combine the inertial navigation solution and the GPS solution requires error models of the INS and GPS systems. Perturbation method is used to linearize nonlinear equations [11, 12]. The $(n \times 1)$ state vector, \mathbf{x} , of the dynamical system is given by [9]:

$$\mathbf{x} = \{\delta \mathbf{r}^n, \delta \mathbf{v}^n, \boldsymbol{\varepsilon}^n, \delta \boldsymbol{\omega}_{iv}^v, \delta \mathbf{b}\} \quad (6)$$

where:

$\delta \mathbf{r}^n$ is the position error (3×1) vector,

$\delta \mathbf{v}^n$ is the velocity error (3×1) vector,

$\boldsymbol{\varepsilon}^n$ is the attitude error (3×1) vector,

$\delta \boldsymbol{\omega}_{iv}^v$ is the angular velocity error (3×1) vector and

$\delta \mathbf{b}$ is a lumped bias error (6×1) vector used to compensate bias and scale factor errors from each IMU accelerometer output.

$\dot{\delta \mathbf{b}} = \mathbf{w} \mathbf{b}$, $\mathbf{w} \mathbf{b}$ is modeled as white noise.

3.3 Kalman Filter Design

The Kalman filter algorithm is used to combine the inertial and GPS measurements in order to compensate the errors included in INS model. Estimates of inertial error states are fed back to the INS algorithms to improve the navigation solution. The error state dynamic equation is given by [14]:

$$\dot{\mathbf{x}} = \mathbf{F} \mathbf{x} + \mathbf{G} \mathbf{w} \quad (7)$$

where, \mathbf{x} is the system state $(n \times 1)$ vector defined in equation (6), \mathbf{F} is the system dynamic $(n \times n)$ matrix, \mathbf{w} is the system noise $(m \times 1)$ vector and \mathbf{G} is the system noise $(n \times m)$ matrix. For these equations the number of states $n = 18$ and the number of measurements $m = 6$. The elements of the error state dynamic equation can be described by [9]:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_r & \mathbf{F}_v & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{F}_r & \mathbf{F}_v & (\mathbf{f}^n) & \mathbf{C}_v^n \mathbf{\Omega}^n - \frac{1}{2\sqrt{2}} \mathbf{C}_v^n \mathbf{J}_2 & \mathbf{0} \\ \mathbf{F}_e & \mathbf{F}_v & -(\mathbf{\Omega}^n) & -\mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\frac{1}{2\sqrt{2}} \mathbf{J}_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (8)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \frac{1}{2\sqrt{2}} \cdot \mathbf{C}_v^n \cdot \mathbf{J}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \frac{1}{2\sqrt{2}} \cdot \mathbf{J}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (9)$$

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_a \\ \mathbf{w}_b \end{bmatrix} \quad (10)$$

The measurement equation is formulated as the difference of position and velocity computed by INS algorithm and given by GPS measurements:

$$\mathbf{z} = \mathbf{H} \mathbf{x} + \mathbf{v} \quad (11)$$

where,

$$\mathbf{z} = \begin{Bmatrix} \mathbf{r}_{\text{INS}}^n - \mathbf{r}_{\text{GPS}}^n \\ \vdots \\ \mathbf{v}_{\text{INS}}^n - \mathbf{v}_{\text{GPS}}^n \end{Bmatrix}_{(6 \times 1)} \quad (12)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{(3 \times 3)} & \mathbf{0}_{(3 \times 3)} & \mathbf{0}_{(3 \times 12)} \\ \mathbf{0}_{(3 \times 3)} & \mathbf{I}_{(3 \times 3)} & \mathbf{0}_{(3 \times 12)} \end{bmatrix}_{(6 \times 18)} \quad (13)$$

$$\mathbf{R} = \text{diag}(\sigma_{\phi}^2, \sigma_{\lambda}^2, \sigma_h^2, \sigma_{v_N}^2, \sigma_{v_E}^2, \sigma_{v_D}^2) \quad (14)$$

here \mathbf{z} is the measurements ($m \times 1$) vector, \mathbf{H} is the observation ($m \times n$) matrix, \mathbf{v} is the measurements noise ($m \times 1$) vector with $\mathbf{v} = (0, \mathbf{R})$, and \mathbf{R} is the observation noise covariance ($m \times m$) matrix and it is given by GPS signal processing.

4. SIMULATION RESULTS

Figure 2 shows a loosely coupled integrated configuration scheme where the GPS data is used as an external sensor [13].

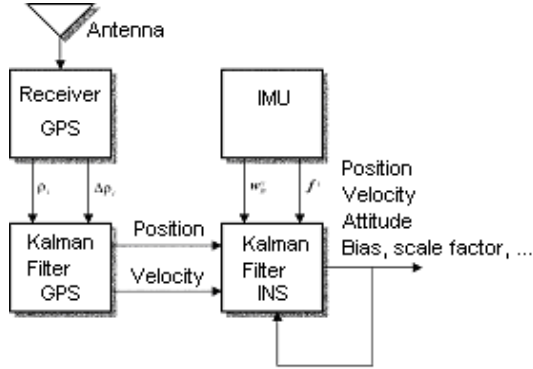


Fig.2. Loosely coupled integration approach

In order to simulate the system, the GPS accuracy measurements values are taken as $\pm 30\text{m}$ for height and $\pm 5 \cdot 10^{-6}\text{rad}$ for latitude and longitude. It is assumed that accelerometers are automotive low-cost type, manufactured in large scale with $20 \cdot 10^3 (\mu\text{g})$ bias, 2000 (ppm) scale factor and $325 (\mu\text{g}/\text{Hz}^{1/2})$ noise. The IMU/INS system is simulated at 100 Hz and corrected at 1 Hz with measurements updates provided by GPS. The following figures show the simulations.

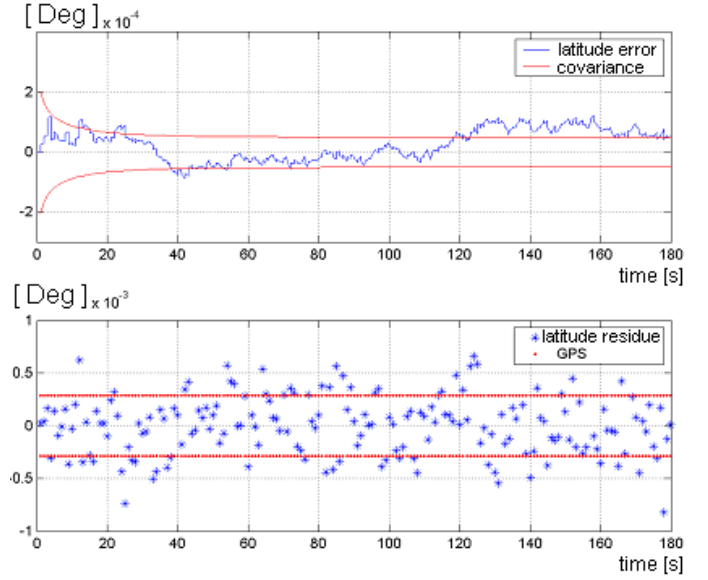


Fig.3. Error and residue for latitude measurements

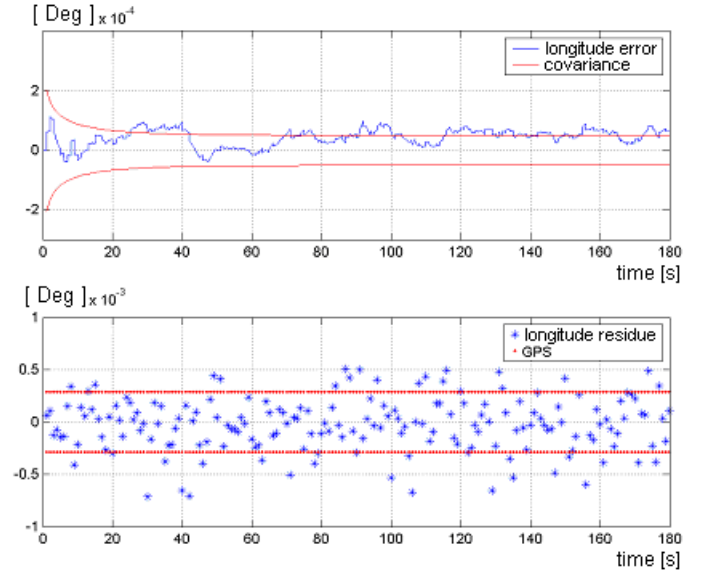


Fig.4. Error and residue for longitude measurements

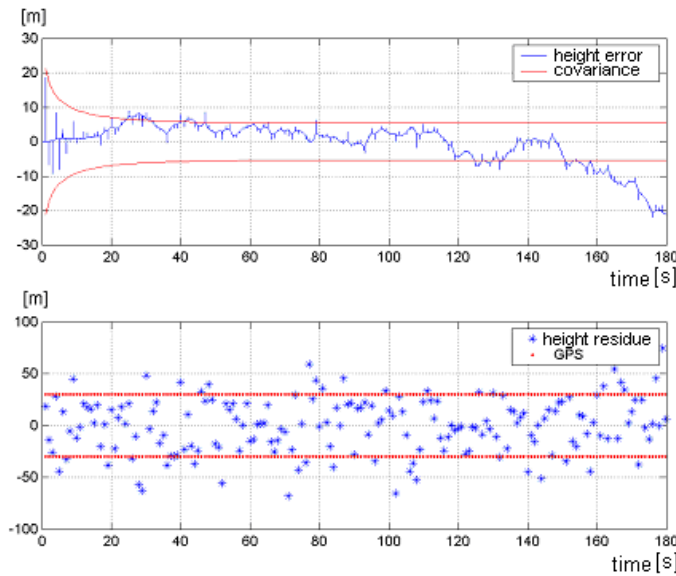


Fig.5. Error and residue for height measurements

Figures 3, 4 and 5 show error curves (nominal value – filtered values) plotted jointly with the errors standard deviations estimated by the filter. The residue curves, $(\mathbf{z} - \mathbf{H} \cdot \bar{\mathbf{x}})$, are plotted with $\pm \sigma_{\text{GPS}}$, the respective GPS measurements standard deviations, and $\bar{\mathbf{x}}$ denotes the propagated state vector.

After a transient period it can be noted that the errors due to the filter are most of the time confined within $\pm 1\sigma$, obtained from covariance, showing a procedure statistical consistency. Also, the residues were distributed around zero deg and confined within $\pm 1\sigma$ from GPS accuracy.

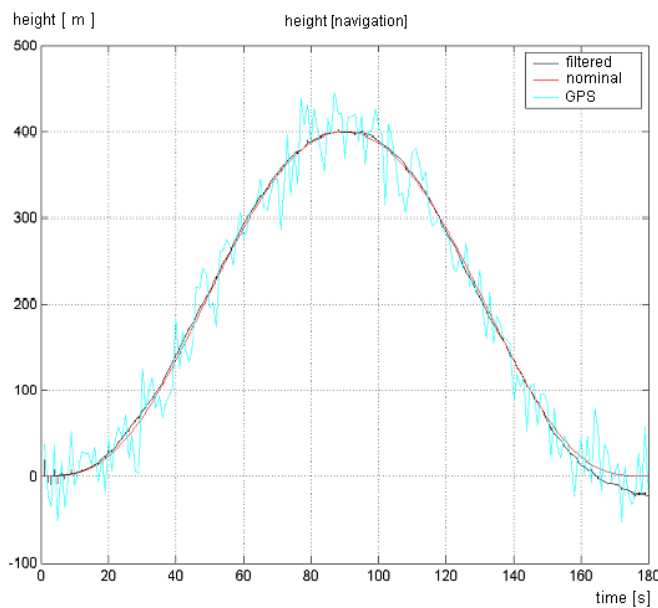


Fig.6. Nominal, filtered and GPS height curves

5. CONCLUSIONS

A non-standard strapdown inertial measurement unit with six accelerometers only and GPS integrated has been analyzed for low cost applications and low-medium performance. The cube type gyroscope free IMU was integrated with GPS data by using the Kalman filter.

An accelerometer lumped error model was presented, where scale factor and bias errors was concentrated in just one parameter modeled as white noise. This model reduces the system state to a 18x1 vector order.

The loosely coupled INS/GPS approach has shown to be sufficient to keep the filtered navigation solution limited by one standard deviation from GPS measurements. Also the simulation results confirm that the integrated navigation solution has better performance than the individuals solutions from GPS and INS.

Besides general navigation applications, the gyro-free IMU has been developed to specific applications as a vibration monitoring device [6], an IMU for tremor sensing in a microsurgical instrument [15] and an IMU to angular and linear velocity estimation for re-entry vehicle [16](Edmundo, passar esta parte para a Introdução).

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