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OPTIMIZATION OF ACTUATORS/SENSORS PLACEMENT AND DERIVATION OF REDUCED ORDER MODELS FOR THE OPTIMAL CONTROL OF FLEXIBLE STRUCTURES

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During the development of control systems for flexible structures, a reduced order controller has to be derived with the requirements of being effective with respect to some "dominant" modes and showing little disturbing effects over the remaining "non-dominant" ones. Closely related to that problem, there exists the necessity of specifying a convenient configuration for actuators and sensors. The purpose of this paper is to present a systematic approach to simultaneously handle order reduction and placement of actuators and sensors. Two methods are presented and compared in this paper. In the first one, actuators and sensors are placed in order to minimize spillover effects and the reduced model is obtained by simple truncation. In the second one, an optimization problem is formulated and the reduced model along with the positioning of actuators and sensors are obtained so as to minimize the difference between the responses of the original and reduced systems. The controllers synthesis in both cases is made using optimal direct output feedback and their performances are verified by checking the reduced controllers against the original model. For the considered application it is found that the two controllers show good performance regarding spillover effects, with the optimal reduced controller exhibiting a little better behavior.

INTRODUCTION

The search of solutions for the control of flexible structures has received great attention in the last years

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due to its widespread application in several engineering systems such as large ships, airplanes and spacecrafts[1]. One of the critical issues of that problem is the necessity of deriving a controller based on a reduced order model for a system which is best represented by a dynamic model of large dimension. As a result spillover may occur, i.e., energy initially directed to some "controlled modes" may be pumped into "uncontrolled" or "unmodelled modes". Another distinguishing feature of the control of flexible structures is that the designer has some "freedom" to place actuators and sensors (A/S) over the structure[2]. Since A/S positioning has a strong impact on controller performance, a method should be sought that can simultaneously treat order reduction and A/S placement.

Standard order reduction procedures usually can be thought as composed by two steps[3]: a)selection of "critical" or "dominant" modes; b)reduced order model evaluation. During the first step, a criterion such as mode controllability, for example, is used to split the original model into dominant and non-dominant parts. In the second step, a reduced order model based on the dominant modes is developed, and a compensation is introduced to take into account the effect of the neglected modes. The performance analysis of reduced order models is often made via the comparison between their response and the response of the original systems[4].

Considering the special features related to the dynamics and control of large flexibe structures, the objective of this work is to present a systematic approach, based on optimal control techniques, that can simultaneously handle the derivation of reduced order models and A/S positioning[5].

Two methods are presented and compared in this paper. In the first one, actuators and sensors are placed in order to minimize spillover effects and the reduced model is obtained by simple truncation of the original model. In the second method, a constrained optimization problem is formulated. The reduced model along with the positioning of actuators and sensors are obtained so as to minimize the difference between the responses of the original and the reduced system to white noise excitations.

The controller synthesis based on the the reduced models, for the two cases, is made using optimal direct output feedback[6] and the performance is verified by checking the obtained reduced controllers against the original model.

To evaluate their behavior with numerically simulated experimental results, the proposed methods are tested adopting a simply supported rectangular plate.

DYNAMIC MODEL ORDER REDUCTION

In order to derive the equations of motion for flexible structures two approaches can be used. For simple structures that can be idealized as beams, membranes, plates, etc., the standard procedure is to write down partial differential equations and use weighted residual methods for discretization. For complex structures, the usual procedure is to use Finite Element Method packages. In both cases, the final result is a system of linear equations written in matrix form, with inertia, stiffness and eventually damping and gyroscopic matrices.

The first step toward order reduction and controller synthesis is to re-write the original system equations in state variable form:

$$dx/dt = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where x is the state vector composed by the modal coordinates and corresponding velocities, u is the input vector dependent on the type of actuators, and y is the output vector which, on the same way, depends on the type of sensors that are used for control purposes.

In Eqs. (1) and (2), the A,B and C matrices have the following structures:

$$A = \begin{bmatrix} O & I \\ -W & -D \end{bmatrix} ; \quad B = \begin{bmatrix} O \\ F \end{bmatrix} ; \quad C = \begin{bmatrix} G & O \\ O & G \end{bmatrix} \qquad (3)$$

where O is the null matrix, I is the identity matrix, $W=diag(\omega_i^2)$ is a diagonal matrix composed by the squares of the eigenfrequencies ω_i, $D=diag(2\zeta\omega_i)$ is also a diagonal matrix depending on the modal damping factor ζ and on the eigenfrequencies, F and G are matrices whose elements are related to the value of the eigenfunctions Phi(r) at the points defined by the position vector r, where actuators and sensors are respectively placed. In the case where actuators and sensors are "dual" (force actuators and displacement sensors for example) and located at the same position (co-located), G is equal to the tranpose of F. It should be stressed that in Eqs. (1) and (2), the B and C matrices are influenced by changing A/S placement.

Taking for example, controllability/observability measures as a criterion, the original system can be separated into two parts:

$$dx_1/dt = A_1 x_1 + B_1 u \qquad (4)$$

$$dx_2/dt = A_2 x_2 + B_2 u \qquad (5)$$

$$y = C_1 x_1 + C_2 x_2 \qquad (6)$$

where x_1 contains the critical modes and x_2 the non-critical ones. In Eqs. (4),(5) and (6) the matrices A_1,A_2,B_1,B_2,C_1 and C_2 have the same structure as the A,B and C matrices previously defined.

The reduced model is written basically taking into account the dominant modes present in x_1, in the form:

$$dx_r/dt = A_r x_r + B_r u \quad (7)$$

$$y_r = C_r x_r \quad (8)$$

In this work two approaches are proposed to obtain the A_r, B_r and C_r matrices. In the first one, A_r is made equal to A_1 and B_r,C_r matrices are "optimized" by changing A/S positions so as to minimize the spillover effect. A "Spillover Index" (SI) is defined involving observability and controllability (O&C) measures of the dominant and non-dominant modes. A configuration of A/S is found such that the O&C of the dominant modes is maximized while the O&C of the non-dominant ones is minimized.

In the second approach, a quadratic performance index is defined which takes into account the difference between the responses of the original and the reduced order models. Since dominant and non-dominant modes are coupled via the output Equation (6), a compensation is introduced by writing the non-dominant modes as a linear combination of the dominant ones:

$$x_2 = Lx_1 \quad (9)$$

A constrained optimization problem is then formulated and the optimal reduced model is obtained via the optimization of A/S positions over the structure (B_r and C_r matrices) and the elements of the L matrix in Eq.(9). Again, the A_r matrix is assumed to be equal to A_1.

Truncated Model

Considering a co-located pair force actuator and displacement sensor, the displacement measured at that position r_a due to an arbitrary excitation F(t) applied by the actuator can be written as:

$$w(r_a,t) = \sum_{i=1}^{n} (Phi_i^2(r_a)/\omega_{di}) \int_0^t F(t)h_i(t-\tau)\, d\tau \qquad (10)$$

where the sum extends over the n considered modes, $\omega_{di}=\omega_i(1-\zeta^2)^{1/2}$ and $h_i(t)=\exp(-\zeta*\omega_i t)*\sin(\omega_{di} t)$. Observing Eq.(10), it can be noted that the convolution integral depends on the type of the applied excitation whereas the constant terms multipling that integral do not. Using this idea, a "Spillover Index" can be defined as follows:

$$SI(r_a) = (\sum_{i=1}^{n_i} Phi_i^2(r_a)/\omega_{di})/(\sum_{j=1}^{n_j} Phi_j^2(r_a)/\omega_{dj}) \qquad (11)$$

where the sum in the numerator of the fraction is related to the dominant modes and the other one, in the denominator, is related to the non-dominant ones. It should be noted that the SI index is associated only with the position in the structure, and if a pair actuator/sensor is placed at a point where SI is maximum, the O&C corresponding to the dominant modes is maximized while the O&C for the non-dominant ones is minimized .

The first reduced order model analyzed in this paper is evaluated taking into account the dominant modes only, without any compensation for the non-dominant ones, being the positions of the A/S specified using the SI index defined in Eq.(11).

Optimal Reduced Model

Different methods have been suggested for the derivation of optimal reduced models[7,5]. The differences in the proposed algorithms are mostly related to the structure selected for the reduced state-space models. In this paper the original method presented by Wilson[7] is modified considering three aspects:

a)the reduced order model structure is mounted over the part of the original model related to the dominant modes (i.e. $A_r = A_1$ and $B_r = B_1$)

b)the contribution of the non-dominant part of the original model to the output of the system is compensated writing the non-dominant modes as a linear combination of the dominant ones (i.e. $C_r = C_1 + C_2 L$)

c)Instead of optimizing the elements of the matrices of the reduced system (A_r, B_r and C_r), an optimzation scheme is developed to determine the best positions for A/S (which affect the B_1, B_2, C_1, and C_2 matrices) and the optimal L matrix.

The first step is to define an error e(t) which is the difference between the responses of the original and reduced systems:

$$e(t) = y(t) - y_r(t) \tag{12}$$

Assuming that both systems are excited by a white noise vector u(t) having the intensity N, the following performance index can be defined:

$$J = \lim_{t \to \infty} E\{e^T(t) Q e(t)\} \tag{13}$$

where E{.} is the expectation operator and $Q=Q^T$ is a positive definite weighting matrix. The objective is to minimize J through the determination of the matrices of the reduced sistem. It is shown elsewhere[7,5] that the performance index J can be re-written as:

$$J = tr[RM] \tag{14}$$

where tr[.] denotes the trace of the matrix and R is the solution of the Lyapunov equation

$$FR + RF^T + S = 0 \tag{15}$$

with the definitions:

$$F = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix} \; ; \; M = \begin{bmatrix} C^TQC & -C^TQC_r \\ -C_r^TQC & C_r^TQC_r \end{bmatrix}$$

$$S = \begin{bmatrix} BNB^T & BNB_r^T \\ B_rNB^T & B_rNB_r^T \end{bmatrix} \tag{16}$$

A necessary condition for optimality is[4]:

$$\partial J/\partial p = 2tr[\partial F/\partial p \; RP] + tr[\partial S/\partial p \; P] + tr[\partial M/\partial p \; R] = 0 \tag{17}$$

where p is any element of the matrices F,S or M, and P is the solution of the associated Lyapunov equation:

$$F^TP + PF + M = 0 \tag{18}$$

Since in this work the parameters to be optimized are the positions of the A/S and the elements of the L matrix, Eq.(17) should be modified. Remembering that r_a denotes the position of A/S in the structure one can write:

$$\partial J/\partial r_a = tr[\partial S/\partial r_a \; P] + tr[\partial M/\partial r_a \; R] \tag{19}$$

where the derivatives of the matrices S and M are present because those matrices are influenced by changing A/S positioning. Using l_{ij} to denote the elements of the L matrix defined in Eq.(9), another relation obtained from Eq.(17) is:

$$\partial J/\partial l_{ij} = tr[\partial M/\partial l_{ij} \; R] \tag{20}$$

The second (optimal) reduced model considered in this paper is evaluated using an algorithm based on Eqs. (14),(19) and (20), satisfying the constraints given by Eqs.(15) and (18).

CONTROLLER SYNTHESIS

In order to compare the two approaches previously mentioned, two controllers are sinthesized and their performances against the original model are verified. Since one is also concerned with the effect of placing sensors, output feedback is considered. It is assumed that the controller should be designed in such a way that only the dominant modes are to be affected. If there is any effect of the controller over the non-dominant modes, this effect should be in the direction of stabilization.

Despite the fact that only necessary conditions are available for this problem, direct output feedback is used in this work. Consider the system equations given by Eqs. (7) and (8), and the control law given by:

$$u = -Hy_r \tag{21}$$

An optimization problem is then formulated by defining a quadratic performance index as follows:

$$J^* = E\{(1/2) \int_0^\infty (x_r^T Q^* x_r + u^T R^* u)\, dt\} \tag{22}$$

where Q^* is a symmetric positive semi-definite matrix and R^* is a positive definite matrix.

Denoting by A_c the system matrix in closed loop, the next equation can be written :

$$A_c = A_r - B_r H C_r \tag{23}$$

Assuming that the A_c matrix in Eq.(23) is "stable", the performance index in Eq.(22) can be written[6] as:

$$J^* = (1/2)\ tr[KX_o] \qquad (24)$$

In Eq.(24) X_o is the covariance matrix related to the initial state $x_r(0)$, assumed to be a random variable, and K is the solution of the following Lyapunov equation:

$$KA_c + A_c^T K + Q^* + C_r^T H^T R^* H C_r = 0 \qquad (25)$$

In order to implement the optimization algorithm, it is necessary to have the derivative of J^* with respect to the matrix H given by:

$$\partial J^*/\partial H = (R^* H C_r - B_r^T K) W C_r^T \qquad (26)$$

where W is the solution of the algebraic Lyapunov equation

$$WA_c^T + A_c W + X_o = 0 \qquad (27)$$

The problem one has at hand at this point is how to specify the weighting matrices Q^* and R^* so as to have the desired closed-loop performance.

Since the equations of motion are written in modal coordinates, the state variables corresponding to each mode are "dynamically" decoupled. This fact suggests that modal control methods could be used, i.e., a controller could be first designed in the modal space where each mode can be independently controlled, and in a second step the real controller could be obtained in the physical world after a coordinate transformation.

Considering that goal, the first task is to write a "modal" performace index where only the i-th mode is taken into account:

$$J_i^* = (1/2) \int_0^\infty (x_{ri}^T\ Q_i^*\ x_{ri} + cf_i^2)\ dt \qquad (28)$$

where Q_i^* is a modal weighting matrix , c is a factor and f_i

is the modal "force". It is well known that the modal forces f_i can be evaluated by:

$$f_i = -(1/c)B_i^T P_i x_{ri} \qquad (28)$$

where P_i is the solution of the algebraic Riccati equation

$$A_i^T P_i + P_i A_i - (1/c) P_i B_i B_i^T P_i + Q_i^* = 0 \qquad (30)$$

Considering that the designer knows where to place closed-loop poles of each mode, it is an easy task to evaluate the modal gains for f_i. The proposed scheme in this work to "specify" the weighting matrices is, knowing the modal gains in advance, evaluate the P_i matrix from Eq.(28) and solve Eq.(30) for the elements of Q_i^*. After that, the reduced system weighting matrices Q^* and R^* are obtained via a transformation[5] which takes into account Eqs.(22) and (28) as well as the input matrix B_r.

NUMERICAL RESULTS

In order to verify the proposed ideas, a physical model of a rectangular plate having its four edges simply supported is considered. The original model is assumed to be composed by the first nine vibration modes, being the lowest four taken as the dominant ones and the next five modes as the non-dominant ones. The modal patterns and frequencies are presented in Fig. 1. In order to specify the input and output matrices, concentrated force actuators co-located with displacement and velocity sensors are utilized.

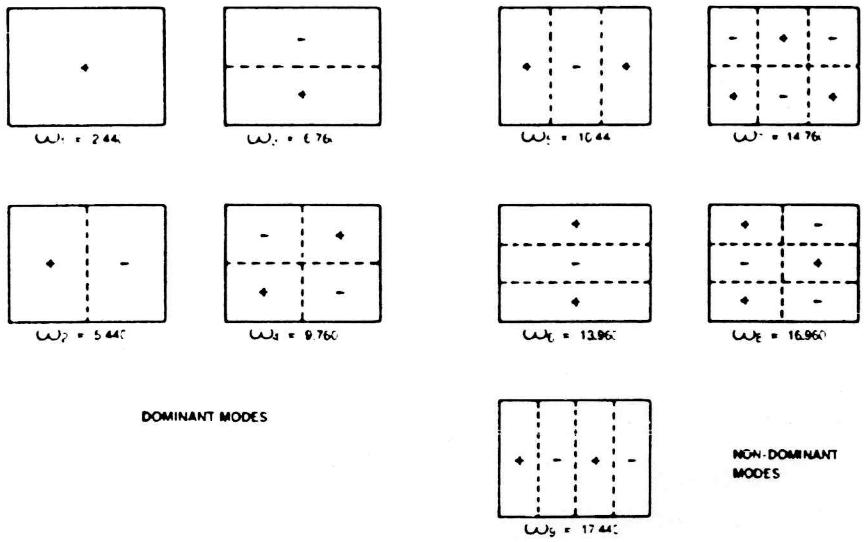

Fig.1 Modal Characteristics of the Original Model

Considering the first method proposed for order reduction, Fig. 2 shows the distribution of the Spillover Index (SI) given by Eq.(11) obtained by varying the position of four A/S symmetrically placed over the plate. That figure illustrates that there is an "optimal" position where the SI is maximum. Placing A/S at those points, the dominant modes are better controlled and the non-dominant ones are less affected.

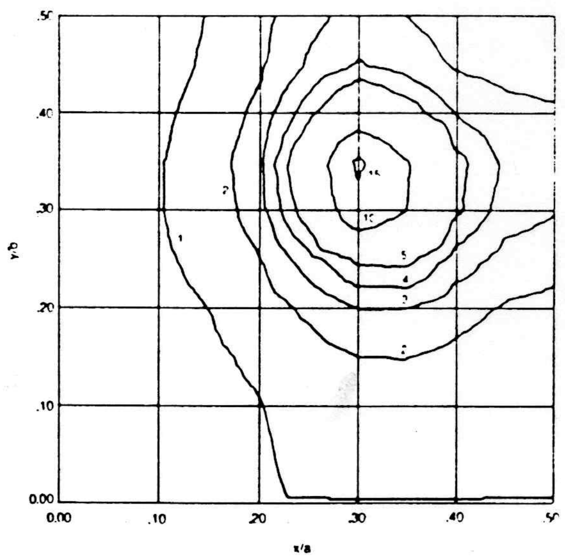

Fig. 2 Distribution of the Spillover Index SI Over the Plate Varying A/S Positioning

For the second approach suggested in this work, Fig. 3 shows the spatial distribution of the quadratic performance index given in Eq. (14). That figure is obtained again by changing the positions of four A/S symmetrically placed over the plate, assuming that the L matrix in Eq.(9) is equal to zero.

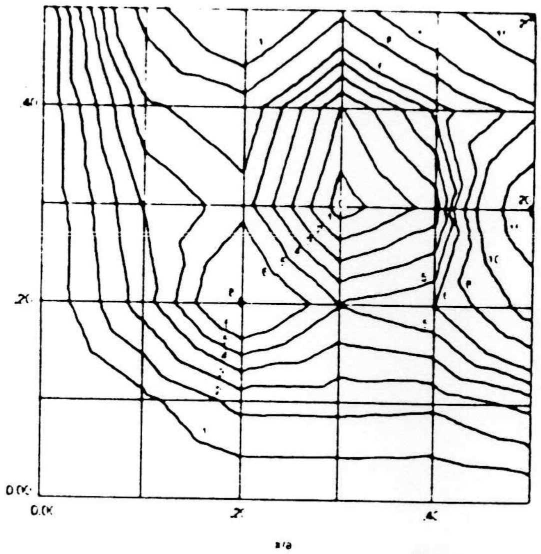

Fig. 3 Distribution of the Performance Index J Due to Changes in A/S Placement Over the Plate

It is interesting to recognize that there is a minimum of J for A/S placed close to the same position shown in Fig.2 for the maximum of SI. These results give support to the hypothesis that the placement of A/S plays an important role for the control of flexible structures. The two different approaches presented in this paper indicate almost the same position as being the best for the placement of these elements.

The complete optimization problem was solved using a gradient method. In order to avoid numerical problems, the optmization procedure was implemented in two steps. In the first one the positons of A/S are optimized keeping the L matrix constant. In the second step the A/S are kept fixed and the best L matrix is sought. Those two steps are repeated until convergence is reached. The final results are presented in Table 1 and 2.

Table 1

OPTIMAL ACTUATORS AND SENSORS PLACEMENT

Actuators/Sensors	x/a	y/b
1	0.2934	0.3281
2	0.7066	0.3281
3	0.2934	0.6719
4	0.7066	0.6719

where x and y denote the position of the pair A/S along the plate and a and b denote its lenght and width. In Table 1, although the final positions for the A/S resulted symmetrical, this symmetry was not imposed in the optimization procedure but is a consequence of the symmetrical boundary conditions of the plate.

Table 2

OPTIMAL L MATRIX

$$\begin{bmatrix} 0.1094 & 0.0000 & 0.0000 & 0.0000 \\ 0.0136 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0479 & 0.0000 \\ 0.0000 & 0.0140 & 0.0000 & 0.0000 \\ 0.0000 & -0.1312 & 0.0000 & 0.0000 \end{bmatrix}$$

It is interesting to notice that only one element at each row of the L matrix presented in Table 2 is significantly different from zero. This fact means that after obtaining the best A/S arrangement, each non-dominant mode is better represented as related to one dominant mode only. Fig. 4 illustrates this result.

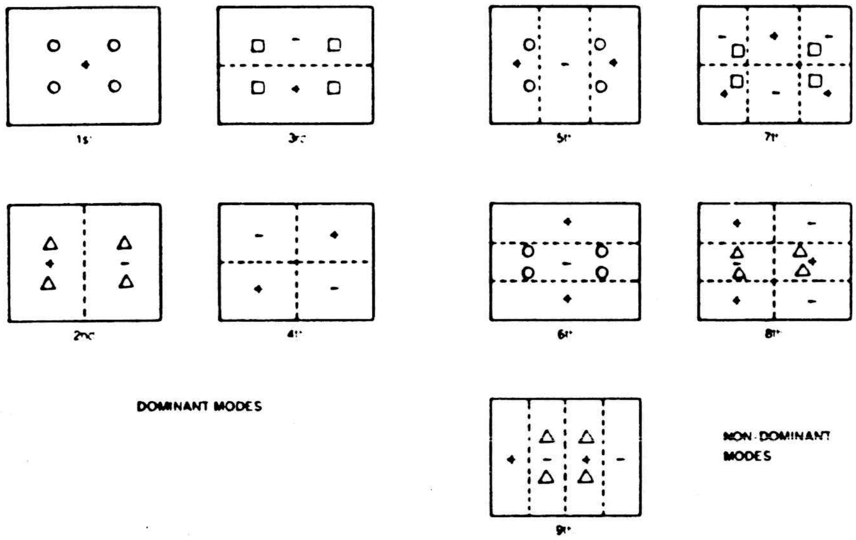

Fig. 4 Relative Positions Between Modal Patterns and Optimized A/S Configuration According to L Matrix Presented in Table 2

Based on the two reduced models previously mentioned, two controllers were synthesized and the results are shown in Fig. 5. That figure presents the desired and obtained positions for the closed-loop poles of the original system.

It can be noticed from Fig. 5 that the proposed scheme for the controller synthesis displays a good performance regarding spillover effects. The poles related to the controlled modes are placed near the "desired" positions while the poles associated to the "non-dominant" modes are not so much affected, being always shiftted to the left direction in the complex plane. Also from that figure, it can be noticed that the performances of the two controllers are very close. Using the value of performance index given by Eq.(24) for the original system in closed-loop as another criterion for comparison, it could be observed that the optimal controller shows a little better behavior[5].

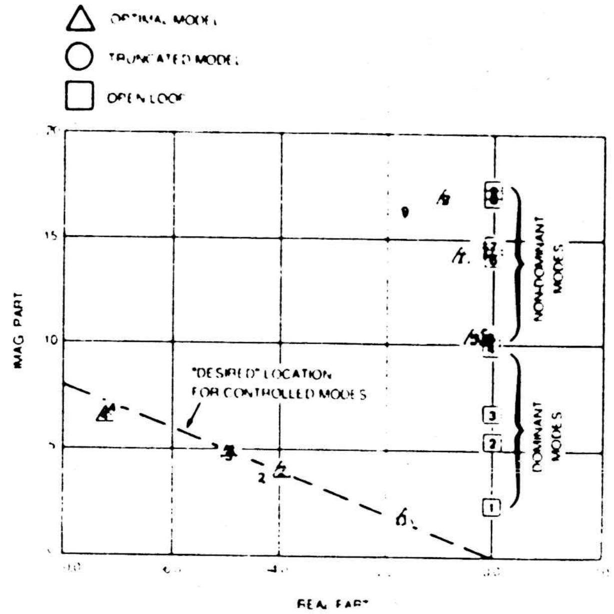

Fig. 5 Positions of the Open and Closed-loop Poles of the Original System in the Complex Plane with Truncated and Optimal Controllers

CONCLUSIONS

When controlling flexible structures, trying to satisfy the requirement of small "spillover" effects, this work was developed with the aim of obtaining a reduced order model to be used in the control synthesis. Due to the high influence of the placement of actuators and sensors (A/S) over the controller performance, two methods were presented and compared taking into account the possibility of changing the positions of those control elements. The first reduced model was obtained by simple truncation of the original model and A/S were placed using a criterion that embodies observability and controllability measures. The second reduced order model was derived using an optimization procedure that led to the minimization of the difference between the responses of the original and reduced order systems. In this case the optimized parameters were the structure of the reduced model and the placement of A/S. Using the two reduced models optimal regulators were synthesized applying direct output feedback and their performances verified with respect to the original model.

Analysing the results obtained from the numerical tests performed, the following remarks can be made:

a)Placement of actuators and sensors has an important role during controller synthesis for a flexible structure.

b)A/S positioning and reduced order model should be treated using a procedure that can take both problems into account, simultaneously if possible.

c)For the considered application, the two controllers obtained by the methods proposed in this work exhibit close performance, being the controller based on the optimal reduced order model a little better.

d)The proposed scheme to synthesize controllers via direct output feedback satisfies the requirement of controlling the dominant modes and having a little stabilizing effect over the non-dominant ones.

Considering the performance of the optimal reduced model, it can be suggested that a natural continuation of this work is to formulate an optimization problem where the structure of the reduced model, the positions of A/S and the gains of the controller could be evaluated using a global approach.

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