SUBOPTIMAL AND HYBRID NUMERICAL SOLUTION SCHEMES FOR ORBIT TRANSFER MANOEUVRES

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ABSTRACT

In this paper the problem of spacecraft orbit transfer with minimum fuel consumption is considered, in terms of selecting, implementing and testing numerical optimal and suboptimal solutions. After a search in the literature and analysis of results available, one selects two schemes of solution to the problem. In the first one the associated optimal control problem is numerically treated by using a direct search approach together with suboptimal parameterized control. In the second one, a hybrid approach is used where the determination of the initial values of Lagrange multipliers (to solve the equations given by the Maximum Principle) is transformed in a direct search problem. In both schemes, the numerical solution of the problem in each iteration is reduced to one of nonlinear programming, which is then solved with the gradient projection method. The spacecraft is suposed to be in Keplerian motion controlled by the thrusters that are assumed to be of fixed magnitude (either low or high) and operating in an on-off mode.

INTRODUCTION

In this paper, from the analyses of the alternatives of solutions available (Ref. 1), results of the implementation and tests of two methods selected to solve the problem of sending a vehicle from one orbit to another with minimum fuel expenditure are shown. The methods can be used either for large orbit transfer (as a geosynchronous satellite launched by the Space Shuttle in a low orbit) or for small orbit correction (as the manoeuvres necessary for stationkeeping of a space station or of a Remote Sensing Satellite).

One of the first solutions to this problem was obtained by Hohmann (Ref. 2), using an impulsive approximation, the so called "Hohmann Transfer". There are many solutions proposed with this type of approximation, like the "Bi-Elliptical Transfer" (Ref. 3) and the "Parabolic Transfer". More recently, a great attention has been given to the more realistic approach, where the thrust is taken as finite, and many researchers have proposed solutions to this case, as, for example, in the works of Tsien (Ref. 4), Lawden (Ref. 5), Ceballos and Rios-Neto (Ref. 6), Rios-Neto and Ceballos (Ref. 7) and Biggs (Refs. 8 and 9).

From the analysis of the alternatives available (Ref. 1), two choices were made:

i) Sub-optimal parametrization;

ii) Optimal control (hybrid approach);

which were explored to develop procedures valid for high or low thrust and large or small transfers.

Numerical results obtained in the calibration and validation of the algorithms developed are presented and compared with similar schemes of solution found in the literature. Together with these results of simulations of the orbit transfer phase of the first Brazilian remote sensing satellite are also presented.

MODEL USED

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusters, whenever they are active. These thrusters are assumed to have the following characteristics:

i) Fixed magnitude (either low or high);

ii) Constant Ejection Velocity;

iii) Either free or constrained angular motions;

iv) Operation in on-off mode.

The solution is given in terms of the time-histories of the thrusters (pitch and yaw angles) and fuel consumed.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the plane of the orbit of the spacecraft) is used as the independent variable.

FORMULATION OF THE OPTIMAL CONTROL PROBLEM

This is a typical optimal control problem, and it is formulated as follows:

Objective Function: -Mf,

where Mf is the final mass of the vehicle and is to be minimized with respect to the control $\underline{u}(.)$, where $\underline{u}(.)$ is any continuous function;

Subject to: Equations of motion, constraints in the state (initial and final orbit) and control (limits in pitch and yaw, forbidden region of thrusting and others);

And given: All parameters (gravitational force field, initial values of the satellite and others)

SUBOPTIMAL METHOD

In this approach (Refs. 1,6,7 and 8), a linear parametrization is used for an approximation of the control (angles of pitch (A) and yaw (B)):

$$A = A0 + A' * (x - xs) \qquad (1)$$

$$B = B0 + B' * (x - xs) \qquad (2)$$

where A0, B0, A', B' are the parameters to be found, x is the instantaneous range angle and xs is the range angle when the motor was turned-on.

With this, there is a set of six variables to be optimized (start and end of thrusting and the four parameters for the angles of pitch and yaw) for each "burning arc" in the manoeuvres. Note that this number of arcs is chosen "a priori".

With control parametrization, the problem is reduced to one of nonlinear programming, which can be solved by several standard methods.

OPTIMAL METHOD

This approach is based on Optimal Control Theory (Ref. 10). First order necessary conditions for a local minimum are used to obtain the adjoint equations and the Pontryagin's Maximum Principle is used to obtain the control angles at each range angle, leading to a "Two Point Boundary Value Problem" (TPBVP), where the difficulty is to find the initial values of the Lagrange multipliers. The treatment given here (Ref. 8 and 9) is the hybrid approach of guessing a set of values, integrating numerically all the differential equations and then searching for a new set of values, based on a nonlinear programming algorithm. With this approach, the problem is again reduced to parametric optimization, as in the suboptimal method, with the difference that the angles parameters are replaced by the initial values of the Lagrange multipliers, as variables to be otimized.

The method proposed by Biggs (Ref. 9) was used here, where the "adjoint-control" transformation is performed and one guesses control angles and its rates at the beginning of thrusting instead of the initial values of Lagrange multipliers. With this, it is easier to find a good initial guess, and the convergence is faster. This hybrid approach has the advantage that, since the Lagrange multipliers remain constant during the "ballistic arcs", it is necessary to guess values of the control angles and its rates only for the first "burning arc". This transformation reduces very much the number of variables to be optimized and, in consequence, the time of convergence.

NUMERICAL METHOD

In order to solve the nonlinear programming problem, the gradient projection method was used (Refs. 11 and 12).

The algorithm was coded in single precision (48 bits) FORTRAN IV, and the calculations were performed at INPE's Burroughs 6800 computer.

SIMULATIONS AND NUMERICAL TESTS

The results of the simulations are as follow.

Table 1

DATA FOR VALIDATION TEST OF ORBIT TRANSFER USING THE SUBOPTIMAL APPROXIMATION (REF. 8)

Orbits	Initial	Final
Semi-major axis (km)	4500	7435
Eccentricity	0.5	0.122
Inclination (degrees)	8.00	2.29
Ascending Node (degrees)	-145.0	Free
Argument of perigee (degrees)	-20.0	Free
True Anomaly (degrees)	170.0	Free

Vehicle data: Initial mass: 11300 kg; Thrust: 60000 N

Other constraints: Thrusting must be completed before true longitude of 35 degrees.

Initial guessed solution (arcs in degrees):

xs = 5; xe = 25; A0 = 0; B0 = 0; A' = 0, B' = 0

Table 2

SUBOPTIMAL METHOD: SOLUTION IN THE LITERATURE (SL) X SOLUTION BY THE ALGORITHM DEVELOPED (SD).

Variable	SL	SD
xs (degrees)	6.7	6.5
xe (degrees)	28	27.7
A0 (degrees)	1.30	0.50
B0 (degrees)	16.00	16.81
A'	-0.017	-0.033
B'	0.007	-0.067
Fuel (kg)	5269.5	5248.9

Table 3

DATA FOR VALIDATION TEST OF ORBIT TRANSFER USING OPTIMAL APPROXIMATION (REF. 9)

Orbits	Initial	Final
Semi-major axis (km)	41904.1	42164.2
Eccentricity	0.018	0.000
Inclination (degrees)	0.688	0.000
Ascending Node (degrees)	-29.8	Free
Argument of perigee (degrees)	7.0	Free
True Anomaly (degrees)	-97.2	Free

Vehicle data: Initial mass: 300 kg; Thrust: 1.0 N

Initial guessed solution (arcs in degrees):

Arc 1: xs = 100; xe = 110; A0 = 180; B0 = -45; A' = 0.5; B' = 0.0

Arc 2: xs = 280; xe = 300

THE BRAZILIAN REMOTE SENSING SATELLITE MISSION

For this mission, two kinds of manoeuvres will be necessary:

i) Transfer phase, where the objective is to send the satellite from the parking orbit to the nominal orbit;

ii) Correction phase, where the objective is to keep the satellite near to the nominal orbit.

The transfer phase will occur, in the worst case, with the data given in Table 4 (Ref. 13):

Table 4

DATA FOR TRANSFER PHASE OF THE FIRST BRAZILIAN REMOTE SENSING SATELLITE MISSION

Orbits	Initial	Final
Semi-major axis (km)	6768.14	7017.89
Eccentricity	0.00591	0.000
Inclination (degrees)	97.44	97.94
Ascending Node (degrees)	67.27	Free
Argument of perigee (degrees)	97.66	Free
Mean Anomaly (degrees)	270.0	Free

Vehicle data: Initial mass: 170 kg; Thrust: 4.0 N

The correction phase will correct the semi-major axis only, and this will occur when its value gets 1.26 km below the nominal value (Ref. 14). Then, a typical manoeuvre has the data shown in Table 5.

Table 5

DATA FOR CORRECTION PHASE OF THE FIRST BRAZILIAN REMOTE SENSING SATELLITE MISSION

Orbits	Initial	Final
Semi-major axis (km)	7016.63	7017.89
Eccentricity	0.000	0.000
Inclination (degrees)	97.94	97.94

Vehicle data: Initial mass: 150 kg; Thrust: 4.0 N

In both phases the fuel used was Hidrazine.

With this statement of the problem, the solutions obtained (Ref. 1) are compared with Hohmann Transfer (Refs. 13 and 14). Initially, the suboptimal method was applied in the transfer phase, with 2, 4 and 8 "thrusting arcs" and no constraints in control. The results are in Tables 6 and 7.

Table 6

TRANSFER PHASE WITH 2 "THRUSTING ARCS"

Arc	xs(deg)	xe(deg)	A0(deg)	B0(deg)	A'	6B'	Fuel(kg)
1	459.8	722.0	11.6	-60.4	0.028	0.500	------
2	963.4	1184.7	17.0	49.8	-0.110	-0.050	14.23

Table 7

TRANSFER PHASE WITH 4 "THRUSTING ARCS"

Arc	xs(deg)	xe(deg)	A0(deg)	B0(deg)	A'	B'	Fuel(kg)
1	498.1	603.4	0.6	-25.7	0.019	-0.053	------
2	1025.4	1125.6	10.4	41.0	-0.159	-0.188	------
3	1590.0	1697.8	3.3	-51.5	-0.009	0.497	------
4	2105.8	2206.6	10.2	40.2	-0.150	-0.183	12.16

Table 8

TRANSFER PHASE WITH 8 "THRUSTING ARCS"

Arc	xs(deg)	xe(deg)	A0(deg)	B0(deg)	A'	B'	Fuel(kg)
1	527.4	576.9	1.1	-16.2	-0.001	-0.052	------
2	1055.3	1105.4	6.6	36.0	-0.151	-0.110	------
3	1622.1	1672.8	2.3	-39.6	-0.004	0.560	------
4	2135.5	2187.6	6.3	35.2	-0.139	-.086	------
5	2327.3	2377.5	1.0	-16.0	0.010	-0.106	------
6	2855.4	2905.7	6.5	35.9	-0.151	-0.110	------
7	3422.2	3473.2	2.2	-39.3	-0.004	0.562	------
8	3935.6	3987.9	6.2	35.0	-0.140	-0.096	11.93

The same manoeuvres were performed with the additional constraints that the control angles must be fixed (A' = B' = 0) with free A0 and B0; and with free B0 and fixed A0 (A0 = 0). The objective was to know how much fuel would be necessary to compensate a more simple implementation of the control device and the constraints to keep some equipments (antennas, for example) pointed toward Earth. The main results are briefly given in Table 8. The value obtained with Hohmann Transfer is about 12.00 kg of fuel. Finally, applying the optimal method to the same transfer phase, the results are shown in Figs. 2, 3 and 4.

Table 10

FUEL EXPENDITURE (KG) FOR THE MANOEUVRES SIMULATED

Method \ Number of arcs	2 arcs	4 arcs	8 arcs
Suboptimal	14.23	12.16	11.93
Suboptimal (A'=B'=0)	21.38	17.05	12.87
Suboptimal (A'=B'=A0=0)	Not found	17.96	13.44
Optimal	13.04	12.09	11.87

For the correction phase, the suboptimal and optimal methods were applied with no constraints in control, and with 1, 2, 3 and 4 "thrusting arcs" applied in different places. The results showed that, in terms of the fuel consumed, there was no improvement when using more than one arc; and for this case the results agrees with those obtained by Hohmann Transfer. So, only this case will be reported.

Table 11

CORRECTION PHASE WITH SUB-OPTIMAL METHOD (1 "THRUSTING ARC")

Arc	xs(deg)	xe(deg)	A0(deg)	B0(deg)	A'	B'	Fuel(kg)
1	0.0	1.56	0.0	0.0	0.0	0.0	47.0

Table 12

OPTIMAL CONTROL (1 "THRUSTING ARC") FOR CORRECTION PHASE

Arc	xs(deg)	xe(deg)	A(constant)	B(constant)	Fuel(kg)
1	0.0	1.56	0.0	0.0	47.0

CONCLUSIONS

By comparing the results obtained by the algorithms developed and those found in the literature (Refs. 8 and 9) it seems that optimal and suboptimal solutions do not exhibit significant differences in fuel consumed, specially when a large number of "thrusting arcs" is used.

The methods have a good numerical behaviour, but they can not be used in real time. Process time (CPU) is short (1 to 3 minutes, in the Burroughs 6800 computer) for simple manoeuvres, but when several constraints and/or "thrusting arcs" are present the process time can be large (more than one hour, in some cases).

Optimization makes no sense when short corrections are involved.

With an increase in the number of "thrusting arcs" the constraints are satisfied in a better way, because there is more controlability. In consequence, the optimal method exhibits more controlability than the suboptimal.

If the initial guess for the number of "thrusting arcs" is larger than necessary, the optimization method can indicate this condition, leading to results where xs = xe for one or more "thrusting arcs".

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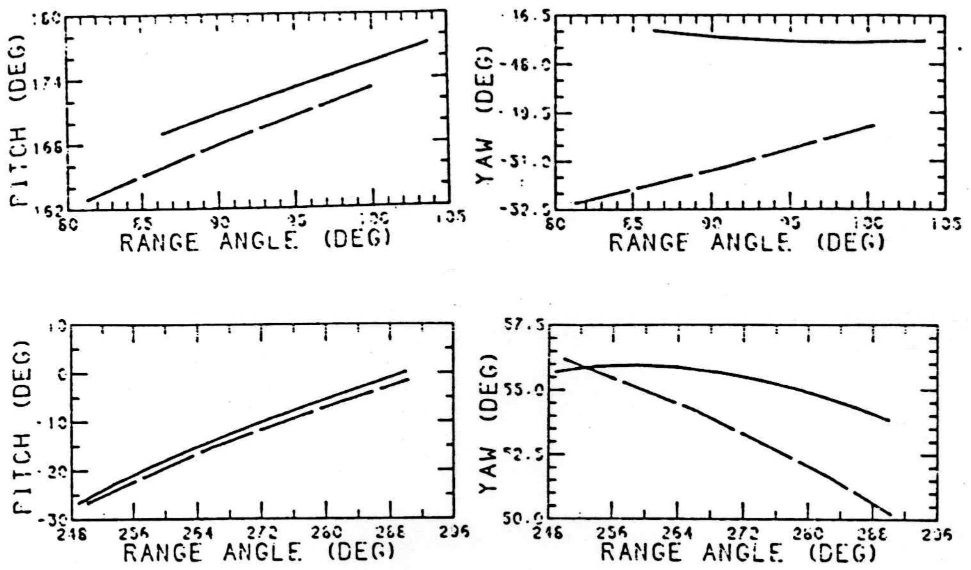
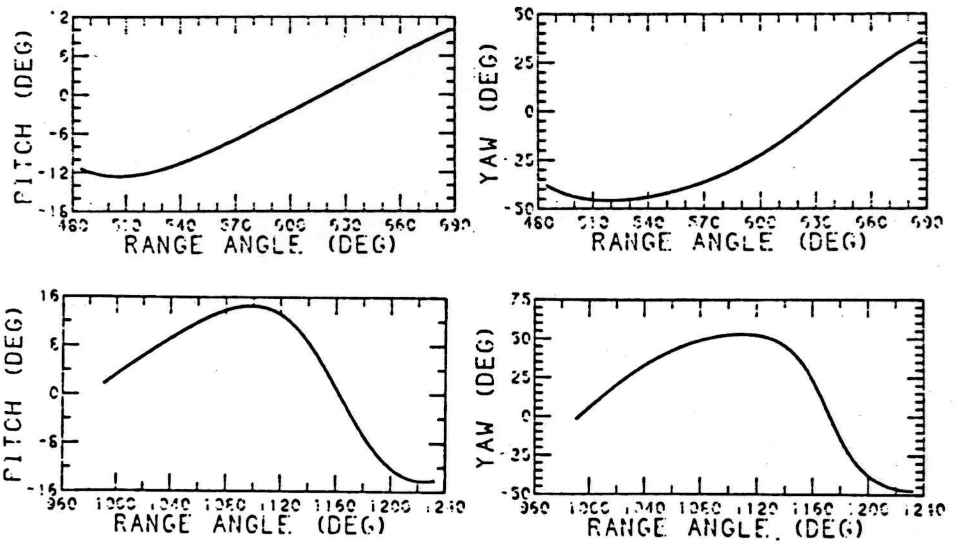


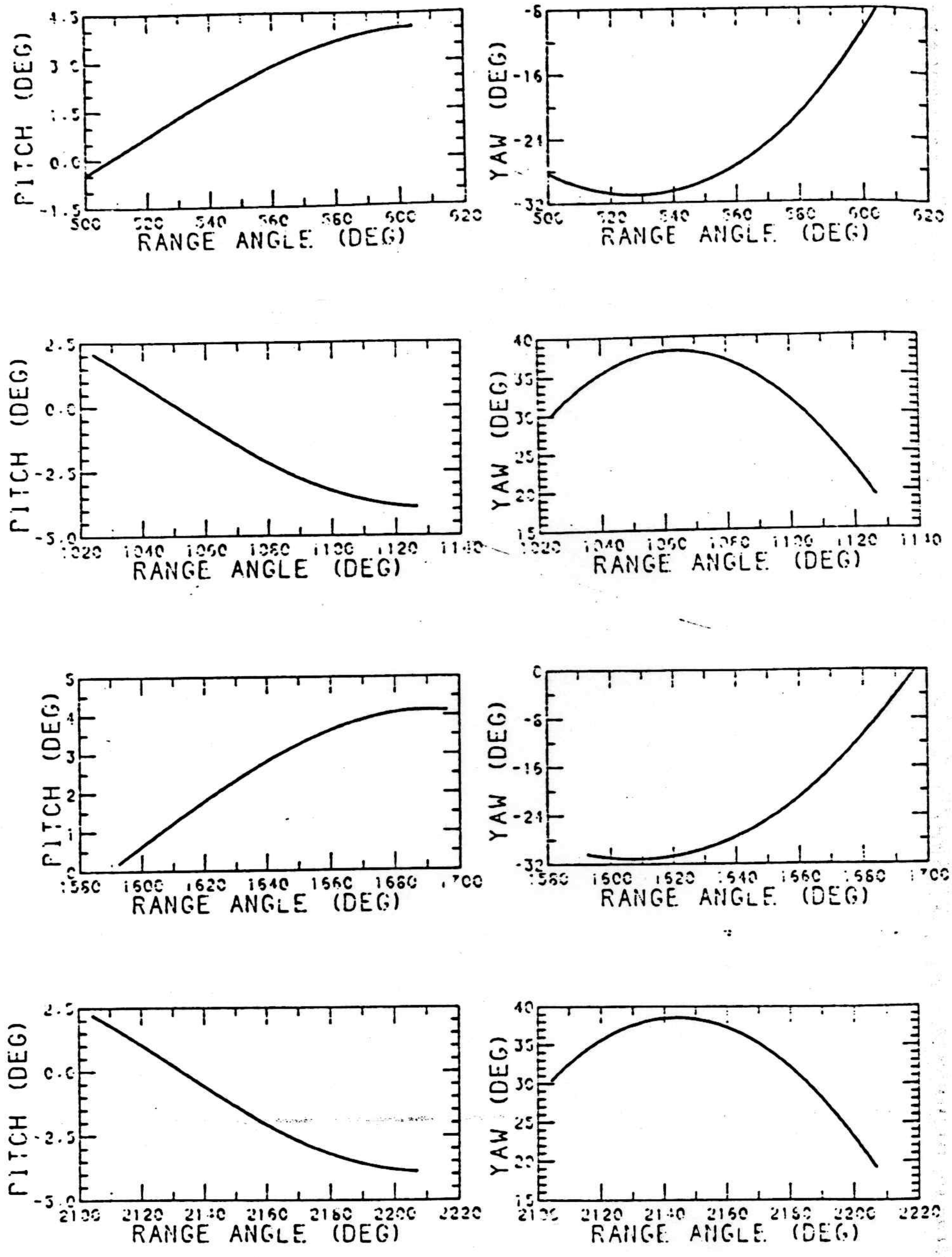
Fuel Consumed (kg): Literature: 5.621; Algorithm: 5.579

Fig. 1 - Solution in the Literature (Dashed Line) X Solution of the Algorithm Developed (Full Line)



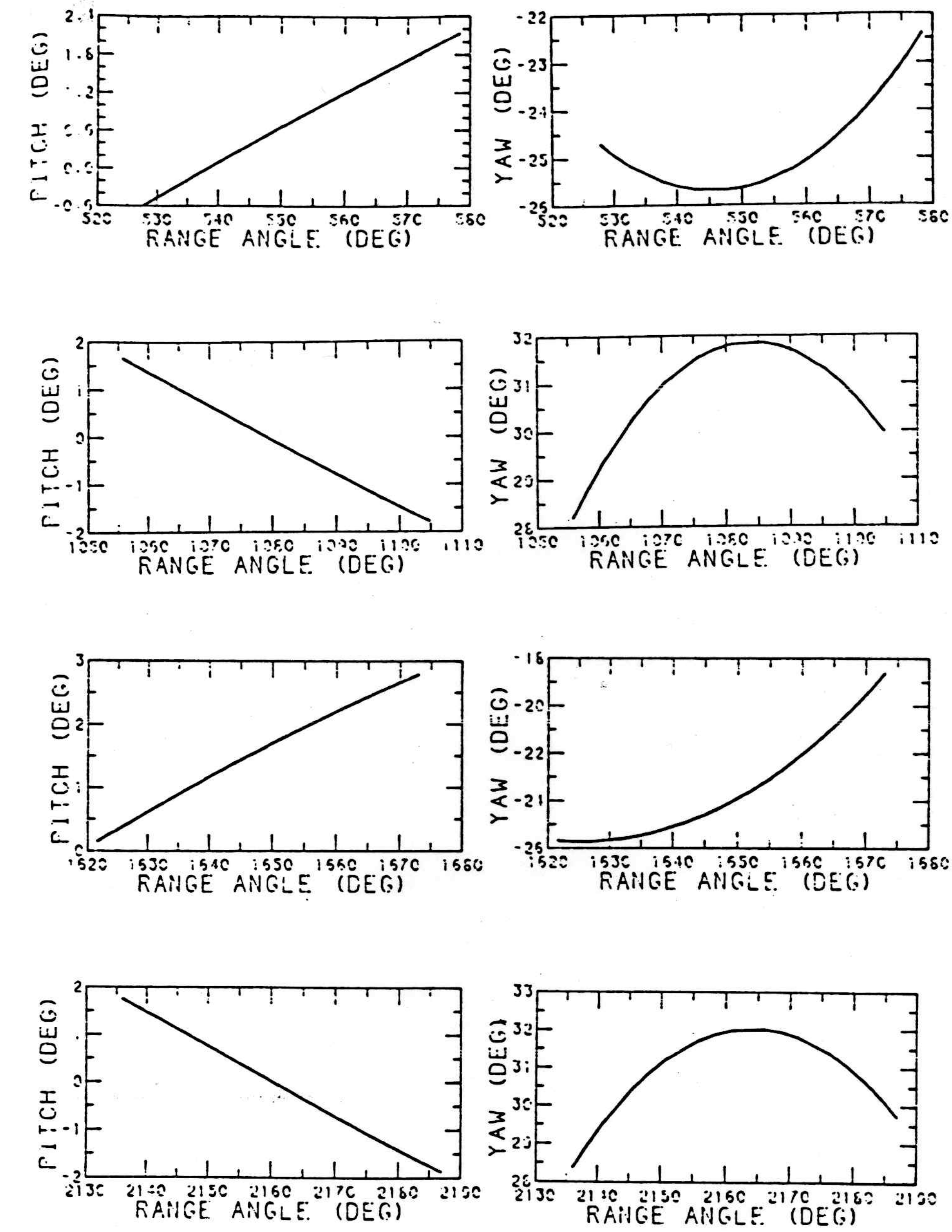
Fuel Consumed: 13.04 kg

Fig. 2 - Optimal Control With 2 "Thrusting Arcs"



Fuel Consumed: 12.49 kg

Fig. 3 - Optimal Control With 4 "Thrusting Arcs"



Fuel Consumed: 11.87 kg

Fig. 4 - Optimal Control With 8 "Thrusting Arcs"

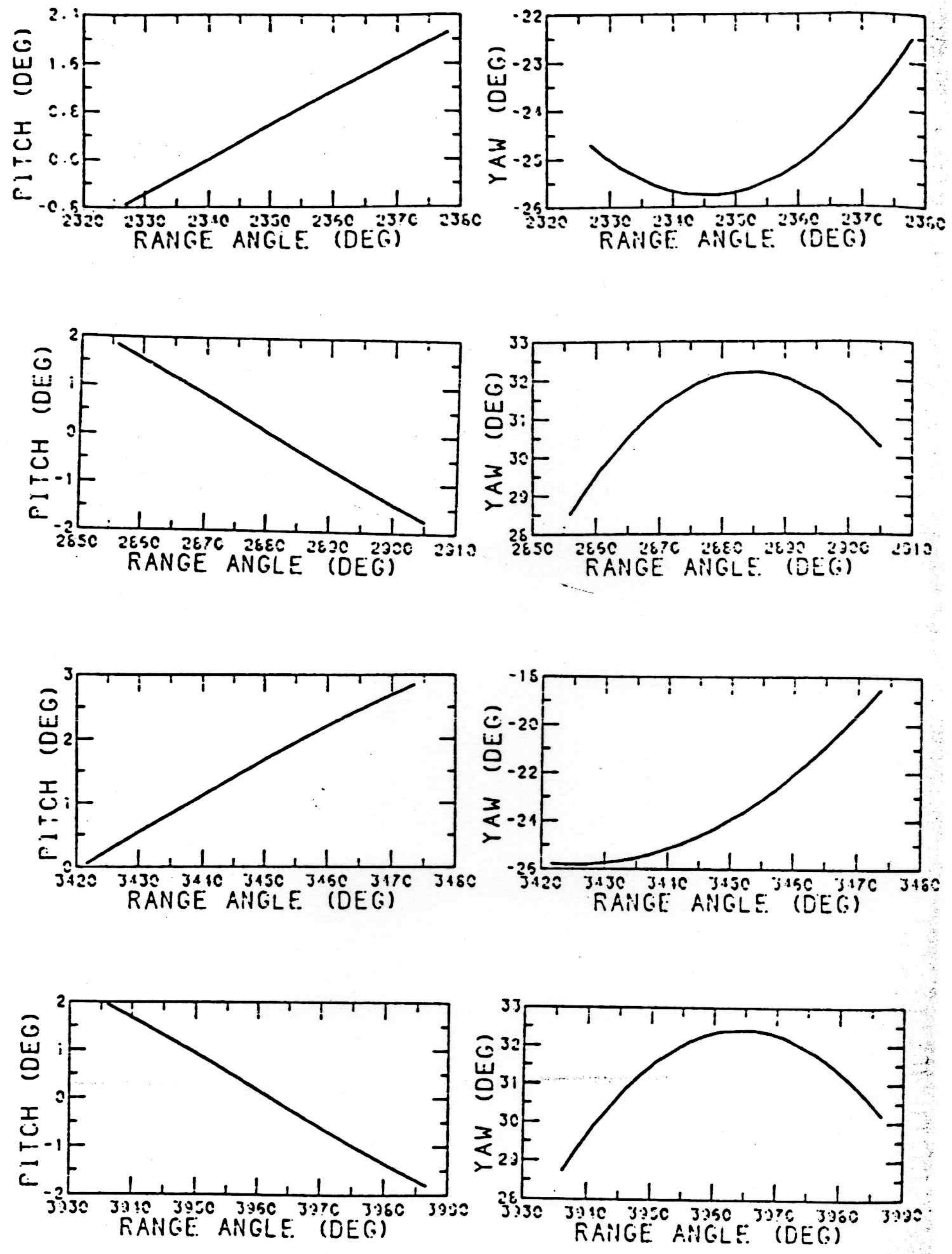


Fig. 4 - Continuation