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TIME-OPTIMAL GEOMAGNETIC ATTITUDE MANEUVERS OF AN AXISYMMETRIC SPINNING SATELLITE

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Abstract—A formulation used to determine the time-optimal geomagnetic attitude maneuvers subject to dynamic and geometric constraints is proposed in this paper. This was obtained by a direct search procedure based on a control function parametrization method, using linear programming to obtain numerical suboptimal solutions by linear perturbation. Due to its characteristics it can be used in small computers and to generate computer programs of general application. The dynamic modeling, the magnetic torque model and the suboptimal control procedure are presented. Simulation runs have verified the feasibility of the formulation thus derived and have shown a notable improvement in performance.

1. INTRODUCTION

The interaction between the onboard coil magnetic moment and the geomagnetic field have been much used to generate control torques. Shigehara [1] developed a switching function, derived from the asymptotic stability condition, for geomagnetic attitude stabilization of a rigid satellite. This control law is applicable for any desired spin-axis orientation and orbital condition. Tossman [2] contributed with an approximate form for a switching function which led to a one-dimensional TPBVP (Two-Point Boundary-Value Problem) approximate optimal maneuver. Junkins *et al.* [3], motivated by the work of Tossman, developed a nonlinear bang-bang switching function using Pontryagin's Principle to solve time-optimal maneuver formulation.

This paper presents a new time-optimal geomagnetic maneuver formulation, using a method for numerical solution of suboptimal control problems, in which the control is taken as a functional dependent on time and a finite number of parameters [4].

The format of this paper is as follows. In Section 2 the satellite dynamic model is described. In Section 3 we present the geomagnetic torque modeling where considerations about spin-stabilized satellite are worked out. In Section 4 we present the suboptimal control procedure used. The model simulated is given in Section 5 illustrating our main result. Concluding remarks are offered in Section 6.

2. SPIN-STABILIZED SATELLITE DYNAMIC MODELING

Consider the inertial frame **OXYZ** shown in Fig. 1, with its origin at the Earth's center **O**, the **Z** axis along the Earth's rotation axis, and **X** and **Y** lie on the equatorial plane of Earth with the axis **X** directed toward the Vernal Equinox γ. The coordinate frame **Oxyz** is associated with the vehicle such that **z** is situated along the axis of symmetry (spin axis) and **x** lies in the inertial plane (**X**, **Y**). Note that the spin axis orientation, in inertial system, is given by $\mathbf{k} = [\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta]$ where α is the right ascension and δ is the declination (see Fig. 1).

Usually, spin-stabilized satellites have passive nutation dampers which serve to dampen nutations rapidly, so the total angular momentum in system **Oxyz** is assumed to be parallel to the spin axis, defined by

$$\mathbf{H} = I_z\dot{\phi}\mathbf{k} \quad (1)$$

where $I_z(>I_x = I_y)$ is the moment of inertia around the spin axis and $\dot{\phi}$ is the spin rate colinear with the axis **k**. Newton's law is written $(\mathrm{d}\mathbf{H}/\mathrm{d}t) = \mathbf{T}$, in which **T** is the torque acting normal to the spin axis, thus

$$\mathbf{T} = I_z[\ddot{\phi}\mathbf{k} + \dot{\phi}(\omega \times \mathbf{k})] \quad (2)$$

where

$$\omega = -\dot{\delta}\mathbf{i} + \dot{\alpha}\cos\delta\mathbf{j} + \dot{\alpha}\sin\delta\mathbf{k} \quad (3)$$

is the angular velocity of coordinate frame **Oxyz**. Combining these equations, we finally obtain the

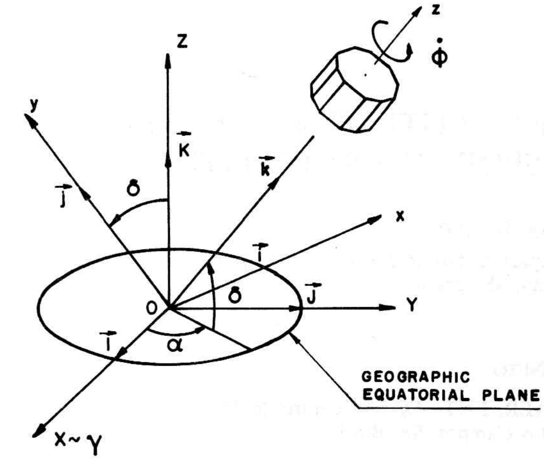

Fig. 1. Reference frames **OXYZ** and **Oxyz**.

approximate rate used for slow maneuvers of the spin axis [5]

$$\dot{\alpha} \cong T_x/(I_z \dot{\phi} \cos \delta) \quad (4)$$

$$\dot{\delta} \cong T_y/(I_z \dot{\phi}) \quad (5)$$

where T_x and T_y are the components of torque in system **Oxyz**.

3. GEOMAGNETIC TORQUE MODELING

The torque that executes the maneuver is generated by the interaction between the coil magnetic moment **m** and the geomagnetic field **B**, given by [6]

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (6)$$

where $\mathbf{m} = p_m m \mathbf{k}$ is the satellite dipole moment with constant magnitude **m** and polarity $p_m(-1, 0, 1)$.

The geomagnetic field is modeled by the centered dipole (see Fig. 2). It is along an axis inclined at $\xi = 11.4°$ to the geographic polar axis. β_α and θ_g are the longitude and the Sideral Time of Greenwich, respectively. The orbital frame $\mathbf{Ox_o y_o z_o}$ is fixed with respect to orbital plane; $\mathbf{z_o}$ is along the orbital normal line and $\mathbf{x_o}$ is along the longitude of the ascending

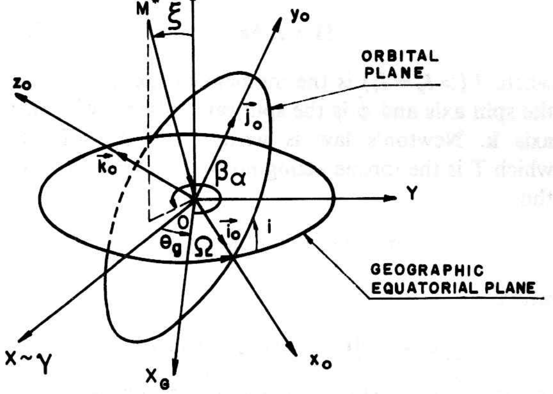

Fig. 2. Orbital coordinate frame.

node. In turn, we used in this work the instantaneous components of control geomagnetic torque [7], in orbital frame, given by

$$\mathbf{T_o} = m(\mathbf{k_o} \times \mathbf{B_o}) \quad (7)$$

where

$$\mathbf{B_o} = [(3M^*y_o/r^3)(\sin\chi \cos\chi) + (M^*x_o/r^3)(3\cos^2\chi - 1), (M^*y_o/r^3)(3\sin^2\chi - 1) + (3M^*x_o/r^3)(\sin\chi\cos\chi), -M^*z_o/r^3] \quad (8)$$

represents the geomagnetic field induction and $\mathbf{k_o}$ is the spin axis orientation, both in orbital frame; $\chi = w_p + v$ and it is the angle between the longitude of the ascending node and the satellite radius vector; w_p is the argument of perihelion; v is the true anomaly; r is the distance from the Earth's center and (M^*x_o, M^*y_o, M^*z_o) are the components of the geomagnetic induction in orbital frame.

It is important to remark upon the following considerations for spin-stabilized satellites with orbits of periods much smaller than 1 day: (i) the spin axis orientation is sensibly invariant during one interval of integration, i.e. one orbit; (ii) to neglect the nodal regression and the apsidal rotation during the same time (the orbit is "frozen"); (iii) to take an average of the geomagnetic dipole component over the time considered (period $\ll$ 24 h).

4. SUBOPTIMAL CONTROL PROCEDURE

A first order direct search procedure [4] for parameter optimization in the numerical solution of dynamical systems control problems is used in this work. The optimal control problem to be treated is to find the control function $u(t)$, in the interval $[t_i, t_f]$, so as to minimize the index of performance and to comply the following constraints

$$IP = IP(x_f, t_f) \quad (9)$$

$$\dot{x} = f(x, u, t) \quad (10)$$

$$M(x_f, t_f) = 0 \quad (11)$$

where x is the $n \times 1$ state vector; $x(t_i)$ and t_i are given; x_f is the final state corresponding to final t_f; u is the $q \times 1$ control vector; and $M(\cdot)$ is the $m \times 1$ vector constraint function of final conditions on state and time: IP is the index of performance.

Supposing that, whenever necessary, the previous extension was made, and if $u(t)$ is substituted by $u(a, t)$ or, in a general form, by $u(a, x, t)$, the problem becomes

$$IP = IP(x_f, a_g) \quad (12)$$

$$\dot{x} = f(x, a_1, a_2, \ldots, a_{g-1}, t) \quad (13)$$

$$M(x_f, a_g) = 0 \quad (14)$$

Table 1. Simulated model

t_i = year 1980, day 350, hour 12, min 0 (GMT)
$M^* = 8.1 \times 10^{15}$ Wb · m^2
$\dot{\phi} = 12$ rpm
$I_z = 8.1$ kg · m^2
Altitude = 1000 km (circular orbit)
i = Inclination = 30°
Ω = Arg. of ascending node = 40.9°
ζ = Orbital period = 100 min

where $x(t_i)$ and t_i are given or defined as the function of the parameters to be optimized; a is the $g \times 1$ vector of the parameters to be optimized and a_g is the final time.

4.1. *Typical iteration associated problem*

From a linear perturbation of eqns (12)–(14), results obtained are:

$$\Delta M = (\partial M/\partial x_f)(\partial x_f/\partial a)\,\Delta a + (\partial M/\partial a_g)\,\Delta a_g \quad (15)$$

$$\Delta IP = (\partial IP/\partial x_f)(\partial x_f/\partial a)\,\Delta a + (\partial IP/\partial a_g)\,\Delta a_g. \quad (16)$$

To satisfy the criterion of getting closer to the suboptimum solution with sufficiently small increments, it is understood that

$$\Delta M = aM, \quad -1 \leqslant a < 0 \quad (17)$$

$$\Delta IP \geqslant b|\overline{IP}|, \quad b < 0 \quad (18)$$

where the condition given by eqn (18), aside from contributing to small increments, means that it is not always possible to get closer to constraint satisfaction and yet to decrease the index of performance.

To choose the problem associated with a typical iteration, which will lead to a scheme for the determination of the search increment, two aspects have to be considered. First, in the limits given by eqn (18), ΔIP should be minimized. Second, to increase convergence speed it is necessary to move along a direction which is close to constraint gradient direction, i.e. a norm of the increment vector Δa should be minimized. Based on these considerations, and from eqns (15)–(18), the associated optimization problem is taken as the minimization of

$$G = \sum_{i=1}^{g} W_i|\Delta a_i| + \bar{W}\,\Delta IP, \quad \bar{W} > 0,\ W_i > 0 \quad (19)$$

subject to

$$(\partial M/\partial x_f)(\partial x_f/\partial a)\,\Delta a + (\partial M/\partial a_g)\,\Delta a_g = aM \quad (20)$$

$$(\partial IP/\partial x_f)(\partial x_f/\partial a)\,\Delta a + (\partial IP/\partial a_g)\,\Delta a_g \geqslant b|\overline{IP}|. \quad (21)$$

To formulate the equivalent problem of minimizing in the usual form of linear programming [4], the following change of variables is made

$$\Delta a_i = s_i - s_{g+i}, \quad s_i \geqslant 0, \quad s_{g+i} \geqslant 0, \quad i = 1, 2, \ldots, g \quad (22)$$

where s_{2g+1} will be introduced to eliminate the inequality sign of eqn (21) and will be used in eqn (19) multiplied by a positive weight to replace $\bar{W}\Delta IP$.

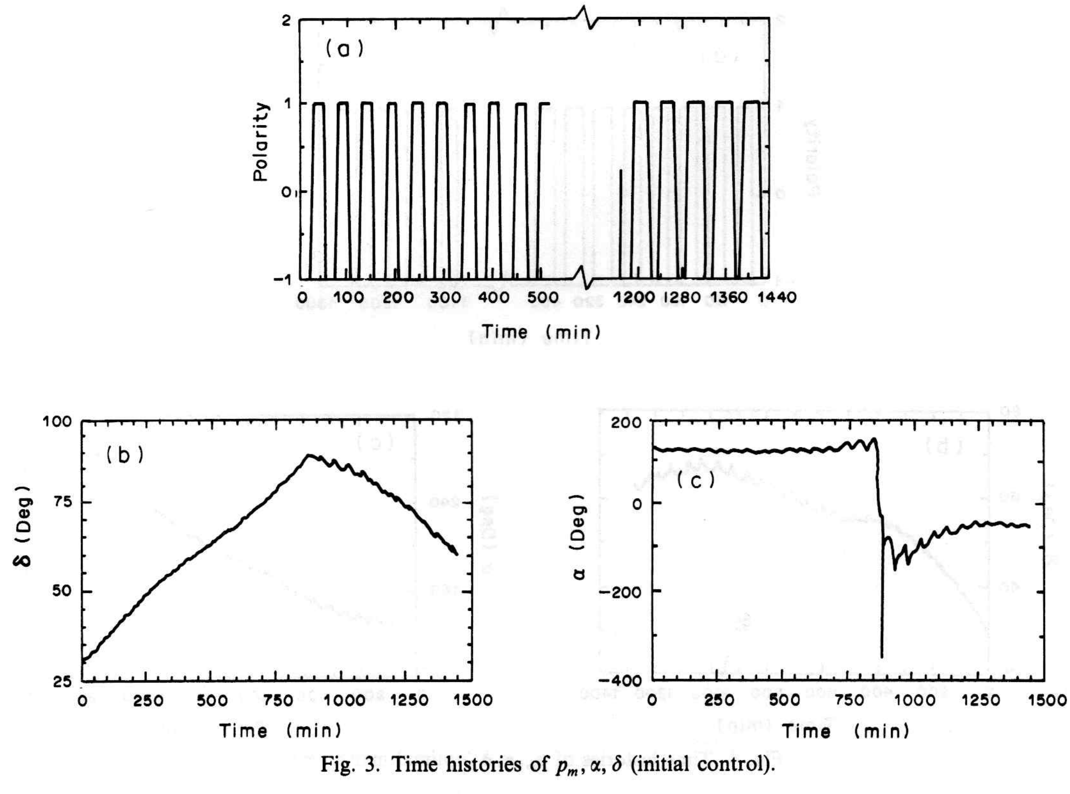

Fig. 3. Time histories of p_m, α, δ (initial control).

5. MANEUVER OPTIMIZATION—FORMULATION AND SIMULATION

The problem formulation consists in suggesting an initial control law for geomagnetic attitude maneuvers and through procedure (Section 4) to obtain a geomagnetic maneuver with an optimum criterion.

As an initial law, we adopted the switching function developed by Shigehara [1] having as a criterion the asymptotic stability condition.

Using the suboptimal iterative procedure, with respect to final time, we obtain the optimal maneuver by parameters (given for the switching points in initial control law) optimization.

5.1. Initial maneuver law

Shigehara [1] developed a switching function to control the spin axis orientation. The desired state $\mathbf{H}_f$, in terms of angular momentum, can be expressed as $\mathbf{H}_f = I_z\dot{\phi}\mathbf{k}_f$, where $\mathbf{K}_f$ represents the desired direction of the spin axis. The difference between $\mathbf{H}_f$ and $\mathbf{H}$ is considered as the error vector, $\mathbf{E} = \mathbf{H}_f - \mathbf{H}$. The objective is to reduce E to zero. In turn, assuming p_m to act in a bang-bang manner [8], the switching function S is defined as

$$S \equiv \mathbf{E}\cdot(\mathbf{k}\times\mathbf{B}) \tag{23}$$

where the control criterion to govern the polarity of p_m is expressed as

$$p_m = +1, \quad \text{when } S > 0$$
$$p_m = -1, \quad \text{when } S < 0. \tag{24}$$

If the polarity of the dipole moment is selected according to the sign of the switching function, the magnitude of error always decreases. Therefore, the desired orientation can eventually be achieved from any initial state.

To illustrate the initial control law, derived above, the same model simulated (see Table 1) by Shigehara will be adopted.

The objective is to find the sequence of switching points in order to maneuver from the initial state ($\alpha_i = 130.9°$, $\delta_i = 30°$) to the desired final state ($\alpha_f = 310.9°$, $\delta_f = 60°$).

The simulation indicates the maneuver final time, $t_f \cong 24$ h, the same result obtained by Shigehara. The history of maneuver is displayed in Fig. 3.

Take note that the singularity problem will happen when the declination will be equal to $\delta = 90°$ [see eqn (4)].

5.2. Time-optimal geomagnetic maneuver

In this section, we used the switching points (given by initial control) as parameters to be optimized through suboptimal procedure. The optimization problem of a generic form [9] to be treated is:

To minimize

$$\underset{p_m(t)}{} \qquad IP = a_g \tag{25}$$

subject to

$$\dot{x}_1 = \dot{\delta} = T_y(p_m(t))/H$$
$$\dot{x}_2 = \dot{\alpha} = T_x(p_m(t))/(H\cos x_1)$$

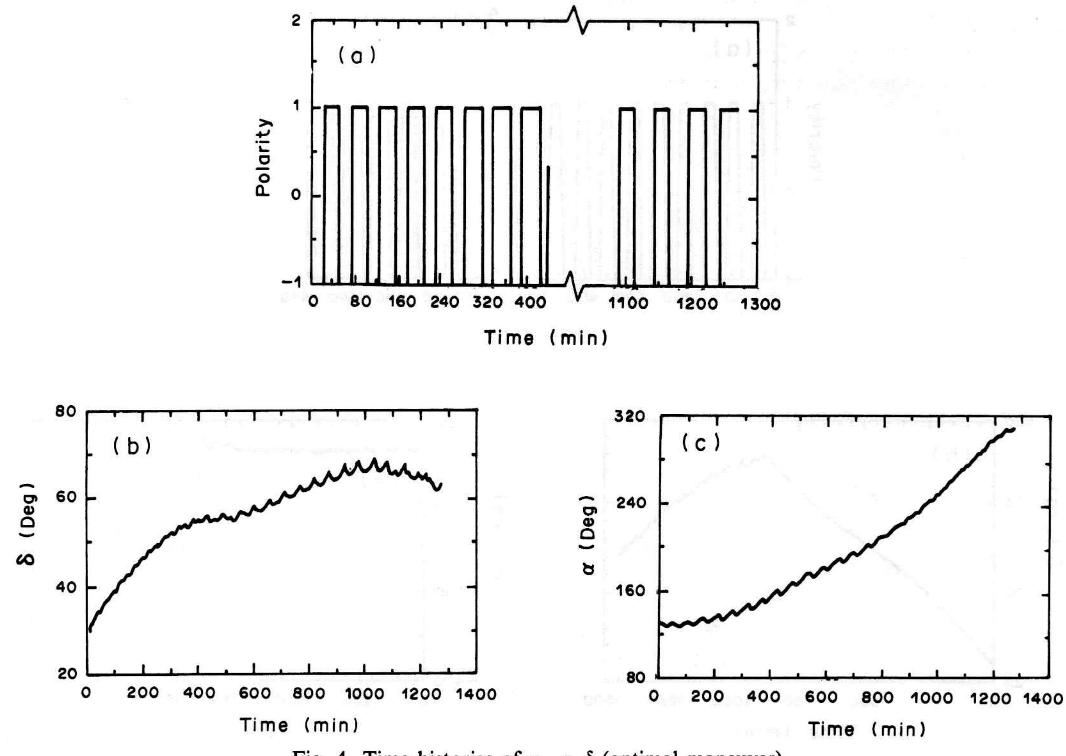

Fig. 4. Time histories of p_m, α, δ (optimal maneuver).

$$M_1(x_{f_1}, t_f) = x_1(t_f) - x_{f_1} = 0$$
$$M_2(x_{f_2}, t_f) = x_2(t_f) - x_{f_2} = 0 \quad (26)$$

where $x(t_i)$ and t_i are given, x_f is the final state corresponding to the maneuver final time t_f and $H = I_z\dot{\phi}$.

After application of iterative procedure, the final maneuver time obtained is $t_f = 21$ h; i.e. it was reduced approx. 12%.

Beyond this, in accordance with Fig. 4, the optimal maneuver obtained is different than the maneuver given by initial control.

Take note that a control pattern weighted halfway between the switching points, such as a pulse, triangular, or sine wave, would give faster control than the rectangular pattern [1].

6. CONCLUSIONS

A new formulation for obtaining optimal time geomagnetic maneuvers by a direct search procedure has been presented. The understanding and facility of implementation of this procedure dispenses with treatment necessity and the explict use (and, therefore, the complexity) of necessary conditions of optimization, making the utilization of the results accessible to non-specialists in the area of optimal control theory of dynamic systems. It is concluded that the procedure is a good choice for optimization of this type of dynamic problems.

To eliminate singularities (present in our work at $\delta = 90°$) and to avoid the lack of numerical precision, it is proposed that one represent the "direction cosine matrix" by using the four Euler parameters (Quaternions).

Based on these factors, and having in view the satellite onboard computation recourses used today, it is suggested that this new formulation be employed to obtain time-optimal geomagnetic attitude maneuver of spin-stabilized satellites.

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