USE OF A VIRTUAL CONTROL APPROACH IN STATE ESTIMATION

A. T. Fleury and A. Rios-Neto

Instituto de Pesquisas Espaciais – INPE, Ministério de Ciência e Tecnologia – MCT, Av. dos Astronautas, 1758, Mail Box 515, 12201 – São José dos Camos – SP, Brazil

Abstract. This work presents the development of a state estimator for dynamic systems exploring the dual function existing between state estimation and optimal control problems. An estimator algorithm similar to the Extended Kalman Filter results to be used in real-time nonlinear systems which can recursively be approximated by linear systems. The original estimation problem is transformed into an equivalent one of virtual control. This control problem is then used to generate and adaptive, locally convergent algorithm where, instead of full state estimation, one has to estimate just a control vector with smaller dimension than,or, in the worst case, the same dimension as the state vector. The control formulation allows different control structures and different convergence acceleration criteria. Two control structures and three criteria are tested in this exploratory phase. The algorithn is implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results are presented for a test case considering critical initial values for the estimator and observations by three Earth stations.

Keywords. State estimation, Adaptive filtering, Kalman filtering, Optimal control.

1. INTRODUCTION

The situation of lack of knowledge about the system dynamic represents a very common problem in state estimation of multivariable systems. In this case, the dynamic model adopted for the estimator is only a crude approximation of reality and, as a consequence, unmodelled effects, mostly for nonlinear systems, can cause divergence of the estimates. In order to avoid divergence, error compensation techniques are usually employed with different versions of the Kalman Filter (e.g. Maybeck, 1979; Jazwinski, 1970; Gelb et al., 1974). These techniques generally explore the information given by the observation residues to either directly estimate the unmodelled effects (e.g.Tapley and Ingram, 1973; Cruz and Rios Neto, 1980: Rios Neto and Cruz, 1985) or to condition the state error covariance matrix to keep the capability of the estimator to extract information from the new observations (e.g. Jazwinski, 1969; Rios Neto and Kuga, 1981, 1982, 1985).

This exploratory work presents a new alternative scheme for the state estimation problem, specially for nonlinear systems. The duality between the functions of estimation and optimal control is explored to transform the original estimation problem into one of tracking the observations with a virtual control. The key idea is that, independently of the dynamic model for the estimator, if the system is completely controllable and observable, one can choose a desirable control action to drive the system towards a region defined by the observations in a finite time interval. Without the need of increasing the number of estimated variables, the observation residues are used to estimate the virtual control necessary to update the estimate of the state. Besides that, the virtual nature of the tracking control allows the possibility of imposing conditions of controllability to better extract the information contained in the observations (Rios Neto and Fleury, 1984; 1985; Fleury,1985). Therefore, the proposed estimator is indicated for situations where there is a great lack of knowledge about the system dynamics, but there is a high local level of information in the observations.

Preliminary tests of the proposed estimator were done under digitally simulated conditions for a critical problem of real-time orbit estimation of a low altitude artificial satellite. Results are shown to be satisfactory for this exploratory phase of procedure.

2. PROPOSED PROCEDURE

The problem to be solved is the state estimation of a multivariable dynamic system of the type:

$$\dot{x} = f(x,t) + f^n(x,t) + \bar{G}(t)\, w(t), \qquad (1)$$

$$y(t_k) = h_k\,(x(t_k),\, t_k) + v(t_k), \qquad k = 1,2,\ldots \qquad (2)$$

where x is the n x 1 state vector; $w(t)$ and $v(t_k)$ are m x 1 and r x 1 zero mean independent Gaussian white noises with the usual hypothesis of $w(t)$ being independent of the past states and $v(t_k)$ being independent of the state, with covariances:

$$E[w(t)\ w^T(\tau)] = Q(t)\ \ \delta(t-\tau)\ ,$$

$$E[v(t_k)\ v^T(t_j)] = R(k)\cdot\delta_{kj}\ , \qquad (3)$$

where $\delta(t-\tau)$ is the Dirac delta function and δ_{kj} is the Kronecker symbol. In this problem, the term $f^n(x,t)$ in Equation (1) represents the unknown part of the dynamical model, which usually cannot be included in the estimator model because of lack of knowledge about the system dynamics.

Consider now a typical discretization interval, $(t_k,\ t_{k+1})$. In the prediction phase of an Extended Kalman Filter (e.g. Jazwinski, 1970), a nominal trajectory is generated by:

$$\dot{\bar{x}} = f(\bar{x},t) \quad ; \quad \bar{x}(t_k) = \hat{x}(t_k|t_k) , \qquad (4)$$

where $\hat{x}(t_k|t_k)$ is the estimate in t_k. Linearizing the system given in Equation (1) around the nominal trajectory and defining:

$$\Delta x(t) = x(t) - \bar{x}(t) , \qquad (5)$$

one obtains a first order approximation of the propagated error as given by:

$$\Delta x(t_{k+1}) = \phi(t_{k+1},t_k)\Delta x(t_k) + \int_{t_k}^{t_{k+1}} \phi(t_{k+1},s)\bar{G}(s)\, w(s)\, ds. \qquad (6)$$

In Equation (6), $\phi(.,.)$ is the state transition matrix associated with the linear system:

$$\Delta\dot{x}(t) = f_x (\bar{x},t)\ \Delta x(t) + \bar{G}(t)\ w(t), \qquad (7)$$

where the subindex x indicates a partial derivation with respect to the state. The propagated error can be regarded as the a priori information in t_{k+1}, which is the information based on previously processed observations:

$$\Delta x(t_{k+1}) = \Delta\hat{x}(t_{k+1}|t_k) + \eta(k+1|k). \qquad (8)$$

Since the nominal trajectory was taken with the initial value in t_k equal to the estimated value at that time, then the propagated estimate in t_{k+1} is necessarily zero. Therefore, if the approximations of Equations (4) and (6) are assumed, there results:

$$\Delta x(t_{k+1}) = 0 + \eta(k+1|k) , \qquad (9)$$

where $\eta(k+1|k)$ is zero mean conditioned on the observations already processed with covariances given by:

$$E[\eta(k+1|k)\ \eta^T(k+1|k)] \triangleq P(k+1|k) = \phi(t_{k+1},t_k)P(k|k)\phi^T(t_{k+1},t_k) + \int_{t_k}^{t_{k+1}} \phi(t_{k+1},s)\ \bar{G}(s)\ Q(s)\ \bar{G}^T(s)\ \phi^T(t_{k+1},s)ds. \ (10)$$

In the proposed procedure the state estimation of the system in Equation (1) is approximated by the estimation of a "virtual control". Defining:

$$\dot{x}_c = f(x_c,t) + G(t)\ u(t) , \qquad (11)$$

the control vector u(t) is to be estimated in order to force the controlled state x_c to be a good approximation of the true state x by tracking the observations given by Equation (2). From an heuristic point of view, the idea is quite simple: Consider a typical interval (t_k, t_{k+1}). Since the model of the estimator is not a good approximation to the real system, within the interval t_k to t_{k+1}, the propagated trajectory will deviate relative to the true one. A control action, based on the observation residues, is then calculated in t_{k+1} to update the estimate of the state vector. This action changes the initial condition for the nominal trajectory in t_{k+1} to a point closer to the true trajectory than the original one. In some sense, this procedure resembles the Extended Kalman Filter, but it must be pointed out that the virtual control scheme requires a number of estimated control components just equal to the number of degrees of freedom of the system (controllability), and also requires a number of observations with enough level of information which guarantees the calculation of the control action in t_{k+1} (observability).

To obtain, in a first approach, the control u(t) in the interval t_k to t_{k+1}, one assumes u as a first order perturbation and takes the controlled trajectory as:

$$\dot{\bar{x}}_c = f(\bar{x}_c,t) , \quad \bar{x}_c(t_k) = \hat{x}(t_k|t_k). \qquad (12)$$

Using the same steps given by Equations (5) and (6):

$$\Delta\dot{x}_c = f_x(\bar{x}_c,t)\ \Delta x_c + G(t)\ u(t), \Delta x_c(t_k) = 0 , \qquad (13)$$

where u(t) is modelled as a step process and calculated to satisfy:

$$\Delta x_c(t_{k+1}) \cong \Delta x(t_{k+1}) , \qquad (14)$$

$$y(t_{k+1}) = h_{k+1}(\bar{x}_c(t_{k+1}) + \Delta x_c(t_{k+1}), t_{k+1}) + v(t_{k+1}). \qquad (15)$$

From Equations (9) and (14) there results:

$$0 = \Delta x_c(t_{k+1}) + \eta(k+1|k). \qquad (16)$$

But from Equation (13) one obtains:

$$\Delta x_c(t_{k+1}) = (\int_{t_k}^{t_{k+1}} \phi_c(t_{k+1},s)G(s)ds)u(t_k) \triangleq \gamma\ (k+1,k)\ u(t_k) , \qquad (17)$$

where $\phi_c(t,t_k)$ is the transition matrix associated with Equation (13). From the linearization of Equation (15) one gets:

$$\Delta y(t_{k+1}) = [\frac{\partial}{\partial x_c} h_{k+1}(\bar{x}_c(t_{k+1}),t_{k+1})]\Delta x_c(t_{k+1}) + v(t_{k+1}) , \qquad (18)$$

where high order terms have been disregarded.

Finally, combining the results of Equations (16), (17) and (18), the following problem of parameter estimation results:

$$0 = \gamma(k+1,k)\ u(t_k) + \eta(k+1|k) , \qquad (19)$$

$$\Delta y(t_{k+1}) = H(k+1)\ \gamma(k+1,k)u(t_k) + v(t_{k+1}), \qquad (20)$$

$$H(k+1) \triangleq \frac{\partial}{\partial x_c} h_{k+1}(\bar{x}_c(t_{k+1}) , t_{k+1}). \qquad (21)$$

Using a Gauss-Markov minimum variance estimator (e.g. Liebelt, 1967 and Maybeck, 1979), estimates of $u(t_k)$ are obtained:

$$\hat{u}(t_k) = P_u(k+1|k)\gamma^T(k+1,k)\ H^T(k+1)\ R^{-1}(k+1)\Delta y(t_{k+1}) \qquad (22)$$

$$P_u(k+1|k) = [\gamma^T(k+1,k)\ P^{-1}(k+1|k)\ \gamma(k+1,k) + \gamma^T(k+1,k).\ H^T(k+1)\ R^{-1}(k+1)\ H(k+1)\gamma(k+1,k)]^{-1}, \quad (23)$$

where R(k+1) and P(k+1|k) are as defined in Equations (3) and (10).

To recover the estimates of the state, one shall combine Equations (17) and (22) to get:

$$\Delta\hat{x}_c(t_{k+1}) = \gamma(k+1,k)\ \hat{u}(t_k) \quad (24)$$

and to take the approximation:

$$\hat{x}(t_{k+1}|t_{k+1}) = \bar{x}(t_{k+1}|t_k) + \Delta\hat{x}_c(t_{k+1}). \quad (25)$$

To recover the covariance matrix of the error in the estimate, it is only necessary to consider Equations (14), (17) and (25) to get:

$$e(t_{k+1}|t_{k+1}) \triangleq x(t_{k+1}) - \hat{x}(t_{k+1}|t_{k+1}) = \Delta x - \Delta\hat{x}_c \quad (26)$$

$$P(k+1|k+1) \triangleq E[e(t_{k+1}|t_{k+1})e^T(t_{k+1}|t_{k+1})] = \gamma(k+1,k)\ P_u(k+1|k).\gamma^T(k+1,k). \quad (27)$$

3. CONTROL MATRIX CORRECTION

In the proposed procedure, the virtual control formulation allows to get some advantages in terms of the estimator dynamical model strucuture. Generally, when one is faced with the problem of state estimation of mechanical systems or other systems governed by second-order differential equations, it is usual to have the matrix $\bar{G}$ in:

$$\dot{x} = f(x,t) + \bar{G}\ w(t)\ , \quad (28)$$

$$\bar{G}^T \triangleq [0_{n/2}\ :\ I_{n/2}] \quad (29)$$

where $0_{n/2}$ and $I_{n/2}$ are the null and the identity matrices of order n/2, respectively. This means that the coefficients in $\bar{G}$ just make the coupling between state variables and noises. In the virtual control context it is possible to define the control matrix G in Equation (11) in order to augment the coupling between estimated controls and state variables and three types of corrections for the control matrix G are proposed.

3.1. DIRECT CORRECTION

The simplest form to define a control matrix with the characteristic described before is to consider:

$$G^T \triangleq [C_p\ I_{n/2}\ :\ C_v\ I_{n/2}] \quad (30)$$

where C_p and C_v are positive coefficients, held fixed during the time propagation (Rios Neto and Fleury, 1984, 1985). Despite the advantage given by the simple form, this correction techniques has the drawbacks of using a trial-and-error method to choose C_p, C_v and of adding the estimated corrections to the state variables with the same sign.

3.2. RESTORATION

The idea of restoration was first used in numerical gradient-like methods to solve optimal control problems. In this case, restoration is used to modify C_p and C_v in Equation (30), thus giving priority of convergence to either forced (velocity type) state variables or non forced (position type) state variables, depending on the system behavior. A decision to change C_p and C_v is made based upon a convergence measure given by estimated errors, e_p for nonforced variables and e_v for the forced ones defined by:

$$e_p(k) = \frac{1}{n/2.\tau_r}\left[\sum_{i=1}^{n/2} P_{ii}(k/k)\right]^{1/2} ;$$

$$e_v(k) = \frac{1}{n/2.\tau_v}\left[\sum_{i=n/2+1}^{n} P_{ii}(k/k)\right]^{1/2} \quad (31)$$

where P_{ii} are elements on the diagonal of P(k/k) and τ_r, τ_v represent the expected standard deviation of the errors between estimated and true variables after convergence. Then e_p, $e_v \cong 1$ indicates convergence to the true trajectory. The procedure is initialized with the Direct Correction and after each $\bar{n}$ steps a test is done. By comparing the estimated errors e_p and e_v with given values (for example, $e_p < 2$ and $e_v < 3$) the estimator makes a decision between: i) changing both coefficients ($e_p > 2$, $e_v > 3$); or ii) changing just C_p ($e_p > 2$, $e_v < 3$); or iii) changing just C_v ($e_p < 2$, $e_v > 3$); or iv) maintaining both coefficients ($e_p < 2$, $e_v < 3$).

With this technique, coefficients C_p and C_v are changed during the time interval of interest. However, the same disadvantages described for the Direct Correction are still present.

3.3. AUTOMATIC CORRECTION

In the automatic correction technique, matrix G is generated at each step inside the algorithm, reinforcing the adaptive characteristic of the procedure. Determination of G is done by solving, step by step, a deterministic linear optimal control problem. This control problem arises if one considers the estimation procedure as that of a controller that must drive the system from a given point in t_k to satisfy the observations in t_{k+1}, as mentioned before. Consider, for (t_k, t_{k+1}), the propagation of the first order perturbation Δx:

$$\Delta x(t_{k+1}) = \phi(k+1,k)\ \Delta x(t_k) + \gamma(k+1,k)\ u(t_k). \quad (32)$$

Assuming that the G matrix is constant in this interval:

$$\gamma(k+1,k) = \int_{t_k}^{t_{k+1}} \phi(t_k,s)\ G(s)\ ds = \left[\int_{t_k}^{t_{k+1}} \phi(t_k,s)\ ds\right] G_k \triangleq \bar{B}_k G_k \quad (38)$$

and using Equation (33) in (32):

$$\Delta x(t_{k+1}) = \phi(k+1,k)\ \Delta x(t_k) + \bar{B}_k G_k\ u(t_k) \quad (34)$$

Consider now that all terms in the right-hand side of Equation (34) are known except the G_k matrix. In t_{k+1}, it is expected that $\Delta x(t_{k+1})$ is calculated to satisfy the observation residues $\Delta y(t_{k+1})$ and this corresponds to minimize a quadratic criterion given by:

$$J = [\Delta y(t_{k+1}) - H_{k+1}\ \Delta x(t_{k+1})]^T[\Delta y(t_{k+1}) - H_{k+1}\Delta x(t_{k+1})]. \quad (35)$$

subject to the constraint given by Equation (34), where

$$G_k^T = [G_{1,k}^T : G_{2,k}^T] = [\mathrm{diag}(g_1, g_2 \ldots, g_{n/2}) :$$

$$: \mathrm{diag}\,(g_{n/2+1}, \ldots g_n)] \qquad (36)$$

There results a linear system to calculate the non null elements of the G_k matrix. If $r<n$ (r is the dimension of $\Delta y(t_{k+1})$) some additional information has to be provided in the criterion to be minimized.

The automatic correction technique has the distinguished feature of providing coefficients for G_k adjusted at each step, with variable signs and magnitude. However, an one-step lag has to be imposed to G_k, since it was assumed that terms of Equation (34) were already known.

4. STATE ERROR FEEDBACK

This approach is in some sense, based on the automatic Correction. The idea is to feedback the estimated state error with an one-step lag, while the observation residues are used to estimate the corresponding control gains to force the estimated trajectory towards the real one. In other words, using the same development given by Equations (11) through (17), the control vector is now approximated by

$$u(t_k) = - C(k)\,\Delta\hat{x}_c(t_k) \qquad (37)$$

where $C(.)$ is a $(m \times n)$ control gain matrix to be estimated in each step. Substituting $u(t_k)$ in Equation (17) there results:

$$\Delta x_c(t_{k+1}) \triangleq \gamma(k+1,k)\,u(t_k) = - \gamma(k+1,k)C(k)\Delta\hat{x}_c(t_k) \qquad (38)$$

where $C(k)$ was taken as

$$C(k) = [C_1(k) : C_2(k)] = [\mathrm{diag}[C_1, \ldots C_{n/2}] :$$

$$: \mathrm{diag}\,[C_{n/2+1}, \ldots, C_n]\,(k) \qquad (39)$$

To recover the estimates of the state and of the covariance matrix of the errors in the estimates, one has just to follow the steps similar to those shown in Equations (24) to (27). However, one has to take into account that linearity of the approximation has to be maintained in order to avoid divergence. In consequence, some limit has to be imposed over the maximum value of the estimated gains $\hat{C}(t_k)$. This is done substituting $\hat{C}(t_k)$ for:

$$\bar{C}_i(t_k) = \alpha_i(k)\;\hat{C}_i(t_k) \quad , \quad i = 1, \ldots, n \qquad (40)$$

where $\alpha_i(k)$ is calculated by:

$$|\alpha_i(k)\;\hat{C}_i(t_k)\;\Delta\hat{x}_{Ci}(t_k)| = \beta|\bar{x}_{Ci}(t_k)| \qquad (41)$$

where, in (41), β is a positive constant coefficient to be tunned for each application.

5. APPLICATION

The procedure was tested under digital simulation (Burroughs B6800) for the case of real time orbit determination of a low altitude satellite with:

zero excentricity, 42^o inclination, 250km altitude, $A/m = 0.00076\ m^2/kg$, area over mass ratio, $C_D = 2.0$, drag coefficient.

To simulate the observations, a true orbit was generated by numerical integration, using a dynamic model including the influence of gravity (up to J_6 zonal and C_{44}, S_{44} tesseral coefficients), atmospheric drag and perturbations of Sun and Moon. The model adopted for $f(x_c,t)$, in Equation (11), only included the gravitational effects up to J_2, thus characterizing a situation of lack of knowledge quite serious in the dynamics of the system. It is shown in Kuga (1982) that the application of the Extended Kalman Filter to the same problem, without any error compensation technique, causes divergence of the estimates. Bearing in mind the approximations taken in the proposed procedure, one must expect some difficulties in dealing with this type of problem. Therefore, error compensation techniques are employed.

5.1. NOISE ADDITION

Whenever necessary and to overcome the ill-conditioning on the state error covariance matrix due to the approximation of Equation (6) for the propagation of the error and due to the nonlinearities in the observations, both $P(k+1|k)$ and $P_u(k+1|k)$ are adapted with the help of an Adaptive State Noise Estimation technique (Rios Neto and Kuga, 1981, 1982, 1985). This technique is the generalization of a procedure by Jazwinski (1969) and consists of adding noise to the system through the state noise covariance matrix Q.

In the results that follow it is shown that for the Restoration and Automatic Correction cases it is possible to use constant levels of state noise; in these cases the technique was only used off line, in the calibration phase, to tune the filters with these constant levels of noise.

5.2. OBSERVATIONS

In the test problem of orbit estimation of a low altitude satellite, observations were generated combining the true orbit data with the location of 3 fictitious symmetric topocentric tracking stations to get, at each 1 second, range and range-rate data, contaminated by white Gaussian noise with standard deviations:

$$\sigma_\rho = 10.0\mathrm{m}; \quad \sigma_{\dot\rho} = 0{,}1\ \mathrm{m/s}.$$

5.3. RESULTS

Results for a test case where critical initial conditions are taken for the proposed estimator are shown in this section. The virtual control $u(t)$ in Equation (11) is a 3×1 vector since it is possible to "control an orbit" using three independent forces, one for each axis. Parameters for analysis are true and estimated position errors, true and estimated velocity errors, given by:

$$\Delta r(k) = \{\sum_{i=1}^{3} [x_i(k) - \hat{x}_i(k)]^2\}^{1/2} \quad k = 1,2,\ldots \quad (42)$$

$$\Delta\hat{r}(k) = \{\sum_{i=1}^{3} P_{ii}(k|k)\}^{1/2} \quad k = 1,2,\ldots \quad (43)$$

$$\Delta v(k) = \{\sum_{r=4}^{6} [x_i(k) - \hat{x}_i(k)]^2\}^{1/2} \quad k = 1,2,\ldots \quad (44)$$

$$\Delta\hat{v}(k) = \{\sum_{i=4}^{6} P_{ii}(k|k)\}^{1/2} \quad k = 1,2,\ldots \quad (45)$$

The estimator is initialized with errors of 1000 m in position and 10 m/s in velocity for each component. The results obtained with the Direct Correction technique are shown in Figures 1 and 2. Coefficients C_P and C_V were chosen equal to 5.0 and 2.0, respectively. Adaptive State Noise Estimation (ASNE) was employed in this case. Figures 3 and 4 show the results obtained with the Restoration Criterion with constant level noise (CLN) addition. Coefficients C_P and C_V were chosen in pairs one among the following: (6.0; 1.0) for poor position convergence; (3.0;1.0) for convergence in position and velocity, and (0.1;2.0) for poor velocity convergence (see Section 3.2);

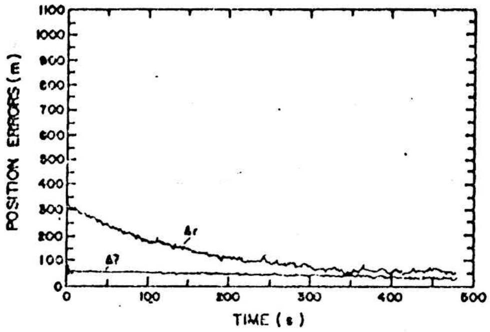

Fig. 1. True and estimated position errors: direct correction, ASNE

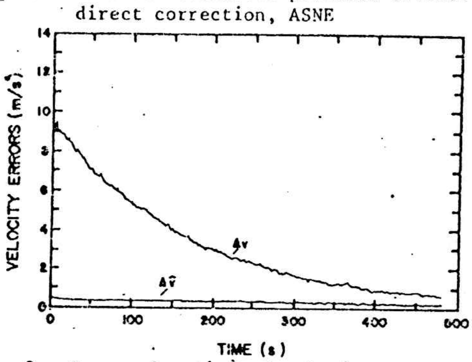

Fig. 2. True and estimated velocity errors: direct correctin. ASNE

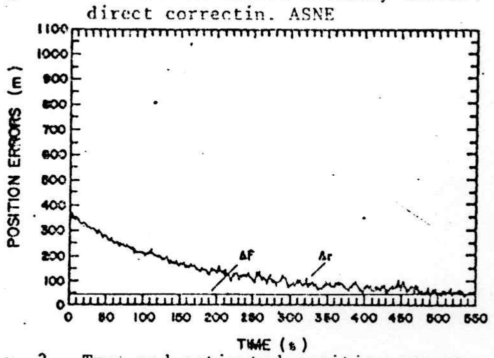

Fig. 3. True and estimated position errors: restoration, CLN.

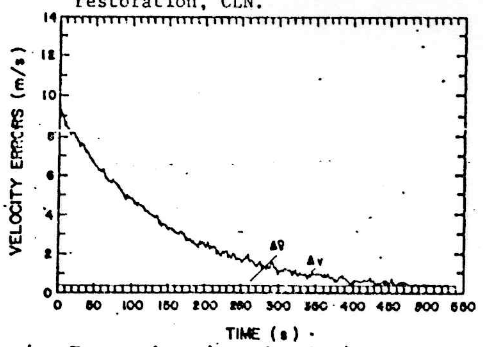

Fig. 4. True and estimated velocity errors: restoration, CLN.

Figures 5 and 6 present the results accomplished with the proposed procedure aided by the automatic Correction and the Adaptive State Noise Estimation technique. Figures 7 and 8 show the results obtained with the use of the Automatic Correction with constant level noise addition.

The results obtained using State Error Feedback and constant level noise addition in the test problems were very similar to those obtained with the Direct C iterion and are not shown. Coefficient β was chosen equal to 10^{-}. Direct correction was also used to augment coupling between the state variables with C_p and C_v equal to 3.0 and 1.0, respectively.

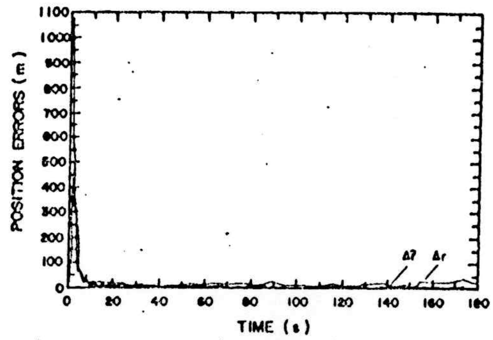

Fig. 5. True and estimated position errors: automatic correction, ASNE.

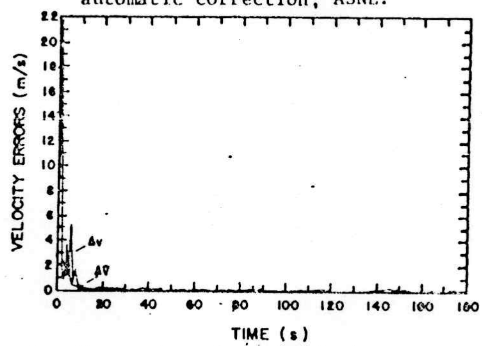

Fig. 6. True and estimated velocity errors: automatic correction, ASNE.

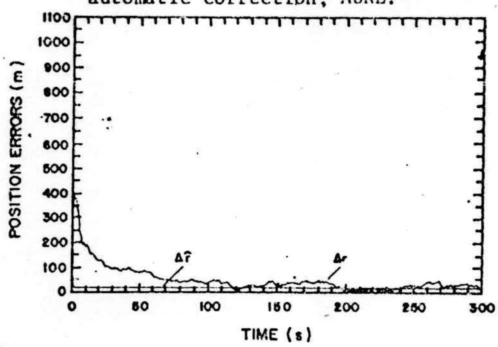

Fig. 7. True and estimated position errors: automatic correction, CLN.

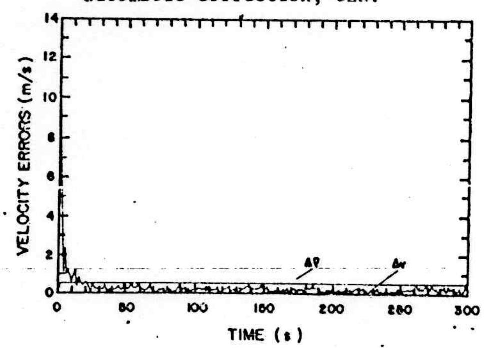

Fig. 8. True and estimated velocity errors: automatic correction, CLN.

The analysis of Figures 1 to 8 shows that good results are accomplished with the use of the Automatic Correction and satisfactory convergence is obtained with the other techniques. In these last cases the estimated errors in the transient phase of convergence are optimistic when compared to the real ones, characterizing the difficulty in introcing adequate noise levels. The time interval for convergence in this transient phase is quite long, and a comparison shows that convergence for positions is better in the case of the Restoration procedure. These characteristics determine that other improvements must be done in the techniques in order to reach a better performance. On the other hand, results obtained with the Automatic

Correction scheme, both with the Adaptive State Noise Estimation technique and with the Constant Level Noise Addition, are comparable to results presented by other techniques usually employed in nonlinear system estimation, such as the Extended Kalman Filter aided by Dynamic Model Compensation (e.g. Kuga, 1982). In this case, a much faster convergence is obtained at the cost of an increment of around 20% in computer processing time, when compared to the other techniques.

6. CONCLUSIONS

A new approach to state estimation of nonlinear systems has been presented, where the duality between control and estimation was explored to transform the estimation problem into one of determining a virtual control. As indicated by the results obtained in this exploratory phase, it is expected to be a valid alternative for the case where a great lack of knowledge in the dynamic exists, but a good level of information is locally provided by the observations.

Up to this point, one only started to explore the possibilities opened by the virtual nature of control in Equation (11). Presently, efforts are being done to further explore these possibilities in order to improve the different estimator versions performance.

REFERENCES

CRUZ, J.J.; RIOS NETO, A. (1980). Estimating the State of Ships in the Presence of Unmodelled Accelerations. American Towing Tank Conference, 19, Ann Arbor, Michigan, USA.

FLEURY, A.T. (1985). Estimadores de Estado de Sistemas Dinâmicos Baseados no Conceito de Dualidade (Dynamical Systems State Estimators Based Upon the Duality Concept). Doctoral Thesis, Escola Politécnica da Universidade de São Paulo, 1985.

GELB, A.; KASPER Jr., J.F.; NASH Jr., R.A.; PRICE, C.F.; SUTHERLAND Jr., A.A. (1974). Applied Optimal Estimation. Cambridge, MA, MIT

JAZWINSKI, A.H. (1969). Adaptive Filtering. Automatica, 5:475-485.

JAZWINSKI, A.H. (1970). Stochastic Processes and Filtering Theory. New York, Academic, (Mathematics in Science and Engineering,64).

KUGA, H.K. (1982). Estimação Adapttiva de Órbita Aplicada a Satélites de Baixa Altitude (Adaptive Orbit Estimation Applied to Low Altitude Satellites) MSc Thesis (Space Sciences). São José dos Campos, INPE, (INPE-2316-TDL/079).

LIEBELT, P.B. (1967). An Introduction to Optimal Estimation. Menlo Park, CA, Addison Wesley.

MAYBECK, P.S. (1979). Stochastic Models, Estimation and Control. V.1, New York, Academic, (Mathematics in Science and Engineering, 141--1).

RIOS NETO, A.; CRUZ, J.J. (1985). A Stochastic Rudder Control Law for Ship Path-Following Autopilots. Automatica, 21(4): 371-384.

RIOS NETO, A.; FLEURY, A.T. (1984). Utilização das Noções de Dualidade no Desenvolvimento de um Estimador de Estado (Use of the Duality Concept in the Development of a State Estimator). São José dos Campos, INPE, (INPE-3288-RPI/110).

RIOS NETO, A.; FLEURY, A.T. (1985). State Estimation Procedure Based Upon the Duality Concept. Presented at the 2nd IASTED International Conference in Telecommunication and Control, Rio de Janeiro.

RIOS NETO, A.; KUGA, H.K. (1981). Estimação Adaptativa e em Tempo Real de Órbita de Satélites Artificiais a Baixa Altitude (Real-Time Adaptive Orbit Estimation of Artificial Satellites at Low Altitude). In: Sixth Brazilian Congress of Mechanical Engineering, Rio de Janeiro. Proceedings. ABCM, v.B., p. 405-414.

RIOS NETO, A.; KUGA, H.K. (1985). Kalman Filtering State Noise Adaptive Estimation. Presented at the 2nd, IASTED International Conference in Telecommunication and Control, Rio de Janeiro.

TAPLEY, B.D.; INGRAM, D.S. (1973). Orbit Determination in the Presence of Unmodelled Accelerations. IEEE Transactions Automatic Control, AC-18(4):369-373.